NINTH GRADED HOMEWORK ASSIGNMENT

Problem 1. There are nine subgroups of the dihedral group $\boldsymbol{D}_8$ of symmetries for the plus-sign. Determine all nine of these subgroups. (Don't forget the whole group and the trivial subgroup.)

Problem 2. Let $\boldsymbol{G} = (G,*)$ be a group with identity element $\boldsymbol{e}$, and suppose that $H \subseteq G$ is nonempty.

PART A: If *H* is a subgroup of $\boldsymbol{G}$, explain why $a * b^{-1} \in H$ for all $a, b \in H$.

PART B: Suppose that $a * b^{-1} \in H$ for all $a, b \in H$.

Part (i): Use the assumption that *H* is nonempty to explain why $\boldsymbol{e} \in H$.

Part (ii): Use Part (i) and the fact that $(b^{-1})^{-1} = b$ to show that if $b \in H$, then $b^{-1} \in H$.

Part (iii): Use Part (ii) to show that if $a, b \in H$, then $a * b \in H$.

Problem 2 gives us what is commonly called the *subgroup test*. It is the simplest test to decide whether or not a nonempty subset of a group is a subgroup of that group:

A nonempty subset *H* of a group $\boldsymbol{G} = (G,*)$ is a subgroup of $\boldsymbol{G}$ if and only if $a * b^{-1} \in H$ for all $a, b \in H$.

Problem 3. Consider the group $\boldsymbol{M}_2$ of all 2x2 invertible matrices under matrix multiplication. Use the subgroup test to prove that the set

$$H = \{A \in M_2 \ : \ \mathrm{Det}(A) = 1\}$$

is a subgroup of $\boldsymbol{M}_2$. State the properties of determinants that you used and what source you used to locate those properties.