TRIGONOMETRY

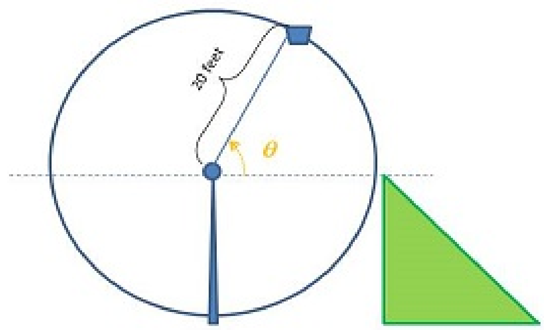
1 Investigation 5 — Sinusoids (Part 1)

A *sinusoid* is a function that has the form

$$y = f(x) = a \sin(\omega x + \theta) + v \qquad \text{or} \qquad y = g(x) = a \cos(\omega x + \theta) + v$$

where a, θ, v, and ω are all constants. In this investigation, we will begin to explore the meaning of these constants and how sinusoids are used to model certain relationships between changing quantities.

The diagram below represents a ferris wheel that has a twenty foot radius. The passengers board the wheel from a platform as shown.



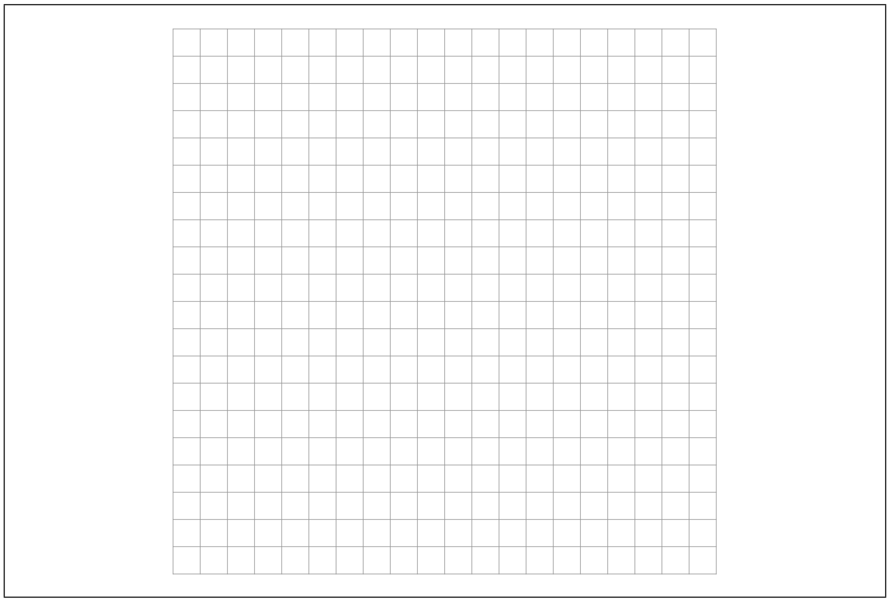
Loomis boards the ferris wheel. The ferris wheel starts to rotate counterclockwise, making one complete rotation every three minutes. As shown in the figure above, let θ be the radian measure of the counterclockwise-oriented angle in standard position formed by the horizontal line passing through the center of the wheel and the metal beam supporting Loomis's gondola.. Let y represent Loomis' vertical distance in feet *above* the platform. (Let negative values of y denote vertical distance below the platform.)

1. Recall the definition of angular speed from Problem 3 of Investigation 2. What is Loomis' angular speed?

2. Fill in the table below.

Number of Rotations	Value of θ	Value of y	Value of $\sin(\theta)$
0			
1/4			
1/2			
3/4			
1			
5/4			
3/2			
7/4			
2			

3. Let f be the function that gives the value of y as a function of the value of θ. Use the information in your table to construct a sketch of the graph of f on the grid below. Be sure to label your axes.



4. Construct the formula for $y = f(\theta)$. Explain how you arrived at your answer.

5. Complete the following table relating the value of θ to the number of minutes that have passed since the ferris wheel started rotating.

Number of Minutes Passed	Value of θ
0	
0.75	
1.50	
2.25	
3.00	

6. Let t be the number of minutes that have passed since the ferris wheel started rotating and let g be the function that gives the value of θ in terms of the value of t. Use your table to construct a formula for the function g.

7. What is the relationship between Loomis' angular speed and the formula you created in Problem 6?

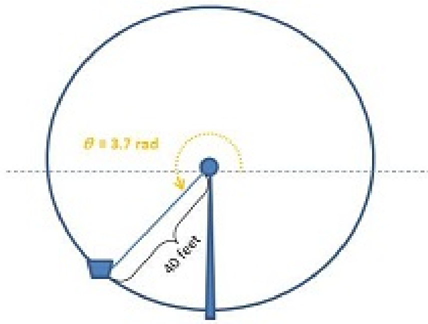
8. Construct the formula for the composite function $f \circ g$. What is the input quantity for this function? What is the output quantity?

9. When Loomis is at the lowest point on the ferris wheel, he is three feet above the ground. Let h represent Loomis' height in feet *above the ground* at any position on the ferris wheel. How could you construct the formula for a function $h = k(t)$ that gives Loomis' height above the ground in terms of the number of minutes that have passed since the ferris wheel started rotating?

10. Imagine a vertical line drawn through the center of the ferris wheel. Construct a formula for the function $x = j(t)$ that gives Loomis' horizontal distance x in feet *to the right* of this line as a function of the number t of minutes that have passed since the ferris wheel started rotating.

11. When Loomis is at the leftmost point on the ferris wheel, he is ten feet to the right of a concession stand located on the ground below. Let d represent Loomis' horizontal distance to the right of this concession stand at any position on the ferris wheel. Construct a formula $d = m(t)$ that gives the value of d in terms of the number t of minutes that have passed since the ferris wheel started rotating.

12. Suppose that Maya boards a different ferris wheel having a forty-foot radius. Maya boards the ferris wheel at the point shown below. Once Maya is onboard, the ferris wheel starts rotating counterclockwise; and Maya makes one complete rotation every half minute.



Part (a) What is Maya's angular speed?

Part (b) As Maya rotates around the wheel, let α be the radian measure of the counterclockwise-oriented angle in standard position formed by the horizontal line passing through the center of the wheel and the metal beam supporting Maya's gondola. Let t represent the number of minutes since Maya boarded the ferris wheel. Construct the linear function $\alpha = p(t)$ that gives α in terms of t. (Hint — what is α when $t = 0$?)

Part (c) Let y represent Maya's vertical distance above the horizontal line. Construct the formula for the function $y = f(t)$ that gives y in terms of t.

13. Suppose a ferris wheel starts rotating when Gerrard gets on board. Suppose further that the function

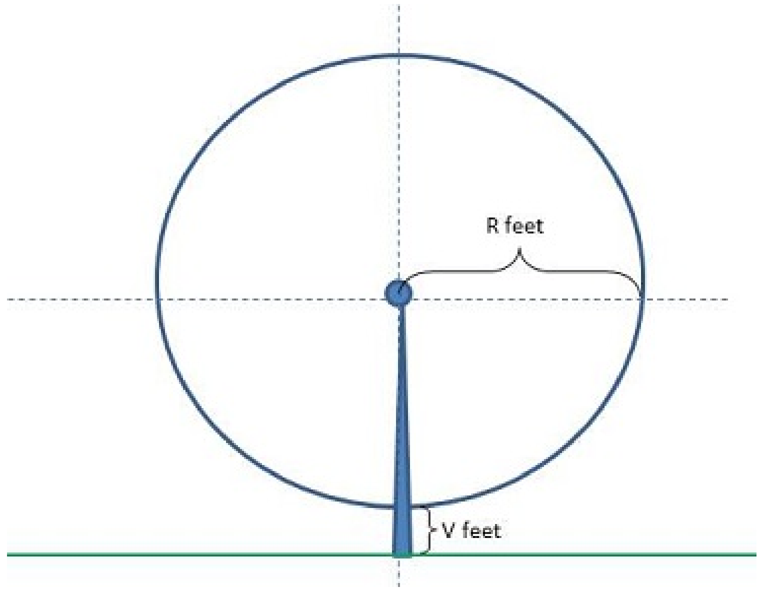
$$h = f(t) = 65 + 59 \sin\left(\frac{4}{5}t + 3\right)$$

gives Gerrard's height h in feet above the ground in terms of the number of seconds t since he boarded.

Part (a) Draw Gerrard's approximate boarding position on the diagram below.

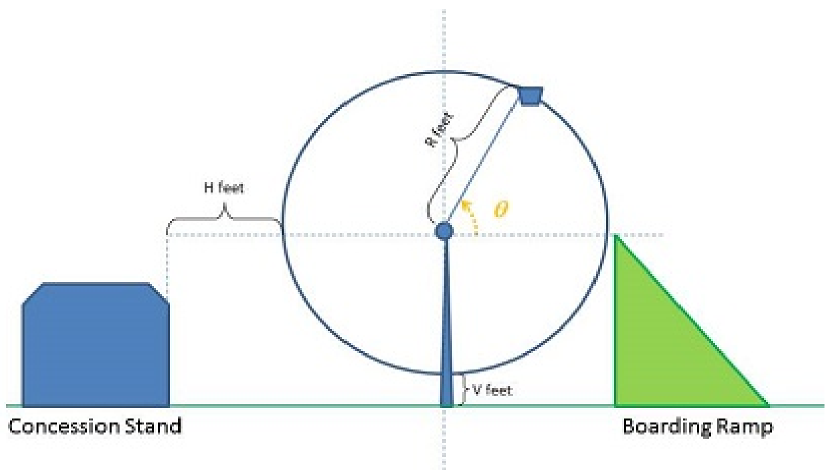
Part (b) What is Gerrard's angular speed as he rotates around the wheel?

Part (c) Determine the value of each constant in the diagram.



Supplemental Problems.

Suppose that Rochelle boards a ferris wheel from a platform as shown in the diagram below. Problems 1, 2, and 3 all refer to this diagram.



1. The ferris wheel starts rotating counterclockwise. What is the Rochelle's angular speed of the if we know

   (a) she completes one full rotation every minute?

   (b) she travels through $\dfrac{\pi}{6}$ radians every half minute?

(c) she travels through 50° every three-quarters of a minute?

2. Suppose Rochelle is V feet from the ground at her lowest point on the ferris wheel. Let t represent the number of minutes since the ferris wheel started rotating and let d represent Rochelle's vertical distance in feet above the boarding platform at any point on the wheel. (Let negative values of d represent distances *below* the platform.) Let h represent Rochelle's height in feet above the ground at any point on the wheel. Construct the formula for a function f that gives d in terms of t and the formula for a function g that gives h in terms of t if we know

(a) Rochelle completes one full rotation every minute, the radius of the ferris wheel is twenty feet, and $V = 3$ feet.

(b) Rochelle travels through $\frac{\pi}{6}$ radians every half minute, the radius of the ferris wheel is ten feet, and $V = 1.5$ feet.

(c) Rochelle travels through 50° every three-quarters of a minute, the radius of the ferris wheel is thirty feet, and $V = 11.8$ feet.

3. Suppose Rochelle is H feet to the right of the concession stand at her leftmost point on the ferris wheel. Let t represent the number of minutes since the ferris wheel started rotating and let d represent Rochelle's horizontal distance in feet to the right of the dashed vertical line through the center of the wheel at any point on the wheel. (Let negative values of d represent distances *left* of this line.) Let h represent Rochelle's distance in feet to the right of the concession stand at any point on the wheel. Construct the formula for a function f that gives d in terms of t and the formula for a function g that gives h in terms of t if we know

(a) Rochelle completes one full rotation every minute, the radius of the ferris wheel is twelve feet, and $H = 5$ feet.

(b) Rochelle travels through $\frac{\pi}{6}$ radians every half minute, the radius of the ferris wheel is fourteen feet, and $H = 20$ feet.

(c) Rochelle travels through 50° every three-quarters of a minute, the radius of the ferris wheel is seventeen feet, and $H = 5.75$ feet.

4. Benny boards a ferris wheel, and the wheel starts rotating counterclockwise once he is onboard. If we let t represent the number of minutes since Benny boarded the ferris wheel, what specific information about the ferris wheel, Benny's angular speed, and Benny's boarding position does the following function give us?

$$f(t) = 29.8 + 25\sin(3.9t + 5.67)$$