In this unit, we will explore one of the most important tools used in algebra. This time, we begin with an example.

Suppose one side of a rectangle has length \( M \) feet, and one side has length \( N \) feet. We know the area of this rectangle is represented by the product \( MN \).

Now, suppose we divide the side of length \( M \) feet into two pieces, one having length \( a \) feet, and the other having length \( b \) feet. Suppose we also divide the side of length \( N \) feet into two pieces, one having length \( x \) feet, and the other having length \( y \) feet.

\[
M = a + b \quad \text{and} \quad N = x + y
\]
Exercise 1

Explain why the rectangle diagram on the previous page tells us

\[(a + b)(x + y) = MN = ax + ay + bx + by\]

The rectangle example motivates the following property relating multiplication of real numbers and addition of real numbers.

**The Distributive Property**

If \(a, b, x, y\) are any real numbers, then the equation

\[(a + b)(x + y) = ax + ay + bx + by\]

will always be valid.

Applying the distributive property to a product is called *expanding* the product.

Exercise 2

Expand the product \((2 + u)(3 + u)\). Combine any like terms in your final answer.

Exercise 3

Expand the product \((u - t)(4 + y)\). Combine any like terms in your final answer.
Exercise 4
Use the rectangle diagram below to determine what expanding the product \((a + b)(x + y + z)\) should give us.

\[ M = a + b \]

\[ N = x + y + z \]

Exercise 5
Use the rectangle diagram on the right to determine what expanding the product \((a + b)x\) should give us.

\[ a + b \]
When we multiply two sums together, we multiply every term in one sum by every term in the other sum and add all of these products together.

**EXAMPLE 1.** Expand the product $(2 - y)(3 + x - t)$.

\[
(2 - y)(3 + x - t) = 2 \cdot 3 + \ldots
\]

\[
(2 - y)(3 + x - t) = 2 \cdot 3 + 2 \cdot x + \ldots
\]

\[
(2 - y)(3 + x - t) = 2 \cdot 3 + 2 \cdot x + 2 \cdot (-t) + \ldots
\]

\[
(2 - y)(3 + x - t) = 2 \cdot 3 + 2 \cdot x + 2 \cdot (-t) + (-y) \cdot 3 + \ldots
\]

\[
(2 - y)(3 + x - t) = 2 \cdot 3 + 2 \cdot x + 2 \cdot (-t) + (-y) \cdot 3 + (-y) \cdot x + \ldots
\]

\[
(2 - y)(3 + x - t) = 2 \cdot 3 + 2 \cdot x + 2 \cdot (-t) + (-y) \cdot 3 + (-y) \cdot x + (-y) \cdot (-t)
\]
EXAMPLE 1 Continued

Once we have multiplied every term in the left-hand sum by every term in the right-hand sum and added all of the resulting products together, we simplify to obtain the expansion.

\[(2 - y)(3 + x - t) = 6 + 2x - 2t - 3y - yx + yt\]

**Exercise 6**
Expand the product \(2t(3 - y - t)\). Simplify your expansion as much as possible.

**Exercise 7**
Expand the product \((3w + 5)(x + w - 1)\). Simplify your expansion as much as possible.

**Exercise 8**
Trevor believes that \((a + b)^2 = a^2 + b^2\). Explain why this is incorrect. What is the correct equation?

**Exercise 9**
Nancy wants to simplify the expression \(2x - 3(x + 4)\). After some work, she comes up with the equation \(2x - 3(x + 4) = 4 - x\). What mistakes did Nancy make? What is the correct equation?
Exercise 10: Expand the product \((x - y)(x^2 + xy + y^2)\). Simplify your expansion as much as possible.

Exercise 11: Solve the equation \(2x - 3(x + 4) = 5\) for \(x\).

Exercise 12: Solve the equation \(4(3 - w) - 2(w - 1) = w\) for \(w\).

Exercise 13: Solve the equation \((t - 1)(2z + 3) = z + 1\) for \(z\). Exclude any values of \(t\) that cause division by 0 in your solution.

Exercise 14: Solve the equation \((t - 1)(2z + 3) = z + 1\) for \(t\). Exclude any values of \(z\) that cause division by 0 in your solution.

Exercise 15: Solve the equation \(m(3x + 1) - x(m - 4) = 0\) for \(m\). Exclude any values of \(x\) that cause division by 0 in your solution.

Exercise 16: Solve the equation \(m(3x + 1) - x(m - 4) = 0\) for \(x\). Exclude any values of \(m\) that cause division by 0 in your solution.