MORE EXERCISES FOR TRIGONOMETRY

The following exercises are meant to help you practice some of the basic skills used when reasoning with trigonometric functions and angle measure. Answers can be found at the end of the worksheet.

Each problem below gives the number n of radius lengths in the arc of a circle cut by an angle whose vertex is centered in that circle. To the nearest hundredth degree, what is the degree measure θ of each angle? (Let "C" denote clockwise rotation, and let "CC" denote counterclockwise rotation.)

(1) $n = 2$ (CC) (2) $n = 4.27$ (C) (3) $n = 22.35$ (CC) (4) $n = 1$ (CC)

(5) $n = 34.9$ (CC) (6) $n = 0.25$ (C) (7) $n = 1.57$ (CC) (8) $n = 12.56$ (C)

In each problem below, α is the degree measure of an angle centered in a circle of radius R. To the nearest hundredth, how many radius lengths are there in the arc of the circle cut by each angle?

(9) $\alpha = 217°$ (10) $\alpha = 315°$ (11) $\alpha = -44°$ (12) $\alpha = -125°$

(13) $\alpha = 495°$ (14) $\alpha = -450°$ (15) $\alpha = 335°$ (16) $\alpha = 60°$

Each of the numbers below is the radian measure α of an angle that is coterminal with the angle having radian measure $\theta = 3.77$ `rad`. Through how many complete rotations must we go (and in what direction) in order to get to get *from* the angle with measure θ *to* the angle with measure α?

(17) $\alpha = 10.053$ (18) $\alpha = -8.796$ (19) $\alpha = 47.751$ (20) $\alpha = -52.777$

(21) Tom is spinning a ball in a circle of radius three feet. When the ball travels through an angle of degree measure $\theta = 770°$, what is the length of the arc the ball travels through?

(22) Landon is walking through an angle of radian measure $\theta = -3.75$ `rad` around a circular track. If he travels through an arc of length 562.5 feet, what is the radius of the track?

(23) Melana is riding on a ferris wheel with a radius of fifty feet. If she travels through an arc of length 588.32 feet in the clockwise direction, what is the radian measure of the rotation angle formed by her starting and ending positions?

(24) Renee is riding on a ferris wheel having radius twenty feet. If she travels clockwise from the position $(20, 0)$ through an arc of length 80 feet, what quadrant will her new position be in?

(25) Richard is riding on a ferris wheel having a radius of 100 feet. If he travels counterclockwise from the position $(0, 100)$ through an arc of length 250 feet, what quadrant will he be in?

(26) Tobias is skiing around a circular track having a radius of 6.7 kilometers. He starts at the position $(6.7, 0)$ and stops to rest at the point $(-3.94, -0.808)$. If he traveled through an arc of length 14.74 kilometers to reach his resting point, did he travel in the counterclockwise or the clockwise direction?

(27) Winnie is running around a circular track and travels 235% of the track radius in the clockwise direction before stopping to rest. What is the radian measure ζ of the rotation angle formed between Winnie's starting and ending position?

(28) Ronald is twirling a ball attached to a string in a vertical circle in front of him with an angular speed of 75 radians per second.

Part (a) What is the radian measure λ of the rotation angle formed by the ball after six seconds have passed?

Part (b) If the string is two feet long, what is the distance the ball has traveled around the circle after six seconds?

(29) A circle of radius ten inches is drawn around an angle in standard position. The initial and terminal sides of this angle subtend 440% of the radius in the clockwise direction.

Part (a) What is the radian measure μ for this angle?

Part (b) What is the approximate value of $\sin(\mu)$ and $\cos(\mu)$?

Part (c) What are the x and y coordinates of the point P where the terminal side of this angle intersects the circle?

Part (d) What is the slope of the terminal ray?

(30) Patricia is walking around a circular track in the counterclockwise direction. She walks 150 radius lengths around the track before stopping to rest.

Part (a) What is the radian measure ϕ of her rotation angle?

Part (b) Assume her rotation angle is in standard position and let $P = (x, y)$ be the point where she stops. What percentage of the radius is x and what percentage of the radius is y?

Part (c) If the track has a radius of 100 feet, how many feet did Patricia walk?

Part (d) Assume that Patricia's starting point is due east of the track center. How far north (or south) of the x-axis is Patricia's stopping point?

In each problem below, a point on the terminal side of an angle in standard position is given. Suppose θ is the radian measure of this angle and determine the exact values of $\cos(\theta)$, $\sin(\theta)$, and $\tan(\theta)$.

(31) $P = (3, 6)$ (32) $P = (-2, -7)$ (33) $P = (4, \sqrt{7})$ (34) $P = (\sqrt{2}, -3\sqrt{5})$

In each problem below, a point on the terminal side of an angle in standard position is given. Suppose θ is the radian measure of this angle and determine the exact values of $\sec(\theta)$, $\csc(\theta)$, and $\cot(\theta)$.

(35) $P = (-4, 2)$ (36) $P = (1, -1)$ (37) $P = (0, 3)$ (38) $P = (-\sqrt{2}, 5)$

(39) Suppose a point P lies on the terminal side of an angle in standard position and suppose that P is a distance of $2\sqrt{6}$ units from the vertex. Let α be the radian measure of this angle. If $\sin(\alpha) = -0.778$, what is the y-coordinate of this point?

(40) Suppose a point P lies on the terminal side of an angle in standard position and suppose that P is a distance of 5 units from the vertex. Let θ be the radian measure of this angle. If $\cos(\theta) = 0.235$, what is the x-coordinate of this point?

(41) Suppose an angle in standard position lies in Quadrant IV. If β is the radian measure of this angle, and if $\cos(\beta) = 0.88$, what is the value of $\sin(\theta)$?

(42) Suppose an angle in standard position lies in Quadrant II. If u is the radian measure of this angle, and if $\csc(u) = 10.5$, what is the value of $\cos(u)$ and $\cot(u)$?

(43) Suppose an angle in standard position lies in Quadrant III. If v is the radian measure of this angle, and if $\tan(v) = 17.9$, what is the value of $\cos(v)$ and $\csc(v)$?

(44) Suppose a point P lies on the terminal side of an angle in standard position at distance R from the vertex, and suppose P is in Quadrant II. Let α be the radian measure of this angle. If we know the point P is 18.87% of R to the left of the y-axis, what is the value of $\cos(\alpha)$ and $\sin(\alpha)$?

(45) Suppose an angle with radian measure β is in standard position and suppose its terminal side has slope $m = -2.11$. Suppose P is a point on the terminal ray of the angle 3.9 feet from the vertex that is 90.4% of this distance above the x-axis. What is the value of $\sin(\beta)$ and $\cos(\beta)$?

ANSWERS

(1) We know that the radian measure of the angle is $\alpha = 2$ radians. Now, α is $\dfrac{2}{2\pi} \approx 0.3183$ of 2π (percentage written as a decimal). The degree measure of this angle will be the same percentage of $360°$. Therefore, if we let θ denote the degree measure, we have $\theta \approx (0.3183)(360°) \approx 114.59°$.

Problems 2 - 8 are worked in the same way.

(2) $\theta = -244.65°$ (3) $\theta = 1280.56°$ (4) $\theta = 57.23°$

(5) $\theta = 1999.62°$ (6) $\theta = -14.32°$ (7) $\theta = 89.95°$ (8) $\theta = -719.63°$

(9) We know that the degree measure of the angle is $\theta = 217°$. Now, θ is $\dfrac{217}{360} \approx 0.6028$ of $360°$ (percentage written as a decimal). The radian measure of this angle will be the same percentage of 2π. Therefore, if we let α denote the radian measure, we have $\alpha \approx (0.6028)(2\pi) \approx 3.79$ radians. Now, the radian measure is the number of radius lengths that can be fit on the arc cut by the sides of the angle, so we know that there will be $n \approx 3.79$ radius lengths on this arc.

Problems 10 - 16 are worked in the same way.

(10) $n = 5.50$ (11) $n = 0.77$ (C) (12) $n = 2.18$ (C)

(13) $n = 8.64$ (14) $n = 7.85$ (C) (15) $n = 5.85$ (16) $n = 1.05$

(17) We know that both α and θ represent counterclockwise rotations. Therefore, since $\alpha > \theta$, we will have to move in the *clockwise* direction to rotate *from* the angle with measure α *to* the angle with measure θ. One complete clockwise rotation has radian measure -2π radians. Observe that

$$\alpha - 2\pi \approx 10.053 - 6.283 \approx 3.77$$

Therefore, it takes one complete clockwise rotation to move *from* the angle with measure α *to* the angle with measure θ.

Problems 18 - 20 are worked in the same way.

(18) Two clockwise rotations

(19) Seven counterclockwise rotations (20) Nine clockwise rotations

Problems 21 - 25 make use of the following facts. Assume the vertex of an angle is centered in a circle of radius R and let S be the length of the arc cut by the rays that determine the angle.

- The radian measure α of the angle is the percentage of the radius represented by the arc length. (In symbols, $\alpha = \dfrac{S}{R}$.)
- The degree measure θ of the angle is same percentage of $360°$ as the radian measure α is of 2π.

(21) Arc length and radius length can only be related through the radian measure of the angle; therefore, we must start by converting the degree measure of the rotation angle into radian measure. Now, $\theta = 770°$ is $\dfrac{770}{360} \approx 2.139$ of $360°$ (percentage written as a decimal). Therefore, the radian measure α will be the same percentage of 2π. That is, $\alpha \approx (2.139)(2\pi) \approx 13.39$ radians. Now, we know

$$13.39 \approx \frac{S}{3 \text{ feet}}$$

Therefore, $S \approx 40.31$ feet.

(22) 150 feet (23) Approximately -11.77 `rad`

(24) Renee begins on the boundary between Quadrants I and IV and rotates clockwise. We know that the radian measure for Renee's rotation angle will be $\alpha = -4$ radians, which is more than half way around the wheel, but not quite three-quarters of the way around the wheel in the clockwise direction. Therefore, Renee must be in Quadrant II.

(25) Richard begins on the boundary between Quadrants I and II. He will rotate through an angle with radian meausre 2.5 radians, which is more than one-quarter of the way around the wheel, but less than half way around the wheel in the clockwise direction. Therefore, Richard will end up in Quadrant III.

(26) Tobias begins on the boundary between Quadrants I and IV, and his resting position is in Quadrant III. If he traveled in the counterclockwise direction, he would have to travel more than half way around the track to reach this point. Now, the circumference of the track is $(2\pi)(6.7) \approx 42.096$ kilometers long. Tobias traveled 14.74 kilometers, and this value is less than half of the circumference. Tobias must have traveled in the clockwise direction.

(27) $\zeta = 2.35\ \texttt{rad}$

(28) (a) $\lambda = 450\ \texttt{rad}$ (b) 900 feet

(29) (a) $\mu = -4.40\ \texttt{rad}$ (b) $\sin(-4.4) \approx 0.9516$ $\cos(-4.4) \approx -0.3073$

(c) $x \approx -3.073$ inches $y \approx 9.516$ inches (d) $\tan(-4.4) \approx -3.096$

(30) (a) $\phi = 150\ \texttt{rad}$

(b) Written as a decimal, x is $\cos(150) \approx 0.6993$ of the radius length, and y is $\sin(150) \approx -0.7149$ of the radius length.

(c) Patricia walked $S = 15,000$ feet.

(d) We know that x is approximately -71.49% of the radius length above (north) of the x-axis. This tells us that Patricia is about 71.49 feet *south* of the x-axis.

(31) We know that the point P is located on the terminal side of the angle at a distance $R = \sqrt{45}$ from the origin. Therefore, we know

$$\cos(\theta) = \frac{3}{\sqrt{45}} \quad \sin(\theta) = \frac{6}{\sqrt{45}} \quad \tan(\theta) = 2$$

Problems 32 - 38 are worked in the same way.

(32) $\cos(\theta) = -\frac{2}{\sqrt{53}}$ $\sin(\theta) = -\frac{7}{\sqrt{53}}$ $\tan(\theta) = \frac{7}{2}$

(33) $\cos(\theta) = \frac{4}{\sqrt{23}}$ $\sin(\theta) = \sqrt{\frac{7}{23}}$ $\tan(\theta) = \frac{\sqrt{7}}{4}$

(34) $\cos(\theta) = \sqrt{\frac{2}{47}}$ $\sin(\theta) = -3\sqrt{\frac{5}{47}}$ $\tan(\theta) = -3\sqrt{\frac{5}{2}}$

(35) $\sec(\theta) = -\frac{\sqrt{5}}{2}$ $\csc(\theta) = \sqrt{5}$ $\cot(\theta) = -2$

(36) $\sec(\theta) = \sqrt{2}$ $\csc(\theta) = -\sqrt{2}$ $\cot(\theta) = -1$

(37) $\sec(\theta)$ UNDEFINED $\csc(\theta) = 1$ $\cot(\theta) = 0$

(38) $\sec(\theta) = -\sqrt{\frac{29}{2}}$ $\csc(\theta) = \frac{5}{\sqrt{29}}$ $\cot(\theta) = -\frac{\sqrt{2}}{5}$

(39) Since $\sin(\alpha)$ is the percentage of the radius represented by the y-coordinate, we know that $y = 2\sqrt{6}\sin(\alpha) \approx -3.81$ units.

(40) $x \approx 1.18$ units

(41) We know that $\cos^2(\beta) + \sin^2(\beta) = 1$. Therefore, we know $\sin(\beta) = \pm\sqrt{1-\cos^2(\beta)}$. Now, since the output of the sine function must be negative for angles in Quadrant IV, we know

$$\sin(\beta) = -\sqrt{1-[0.88]^2} \approx -0.47$$

(42) $\cos(u) \approx -0.995 \quad \cot(u) \approx -10.47$

(43) There are several ways to approach this problem. One way would be to let $P = (x, y)$ be a point on the terminal side of the angle with measure ν and observe that

$$\frac{y}{x} = \tan(\nu) = 17.9 = \frac{-17.9}{-1}$$

so that we may take $x = -1$ and $y = -17.9$. (Remember, the angle lies in Quadrant III where both x and y are negative.) This tells us that the point P is a distance $R \approx 17.93$ from the origin. Therefore,

$$\cos(\nu) = \frac{x}{R} \approx -0.558 \qquad\qquad \csc(\nu) = \frac{R}{y} \approx -1.001$$

(44) $\cos(\alpha) = -0.1887 \;\; \sin(\alpha) \approx 0.982$

(45) $\sin(\beta) = 0.904 \;\; \cos(\beta) = -0.428$