Homework Assignment 5

(5) Is the binary function $f : \mathbf{R}^2 \longrightarrow \mathbf{R}^+$ one-to-one or onto?

Solution. This function is neither one-to-one nor onto. To see that it is not one-to-one, observe that $f(1,1) = f(-1,-1)$. To see that it is not onto, observe that $\mathtt{Pre}_f(1/2) = \emptyset$.

(6) Consider the binary function $f : \mathbf{Z}^2 \longrightarrow \mathbf{Z}$ defined by $f(x,y) = 3x + 2y$. Show that this function is not one-to-one but is onto.

Solution. Note that $f(0,3) = f(2,0)$; hence this function is not one-to-one. Now, since 3 and 2 are relatively prime, we know there exist integers a and b such that $1 = 3a + 2b$. Thus, for any integer y, we know that $y = 3(ay) + 2(by)$. It follows that $(ay, by) \in \mathtt{Pre}_f(y)$; and we may conclude that f is onto.

(8) Show that the functions below are both left inverses for $f : \mathbf{Z}^+ \longrightarrow \mathbf{Z}^+$ defined by $f(n) = 2n$.

$$g(k) = \begin{cases} k/2 & \text{if } k \text{ is even} \\ k & \text{if } k \text{ is odd} \end{cases} \qquad h(k) = \begin{cases} k/2 & \text{if } k \text{ is even} \\ 3k & \text{if } k \text{ is odd} \end{cases}$$

Solution. First, note that the output of f is always even. Therefore we know that $f(n)/2 \in \mathbf{Z}$; in fact, $f(n)/2 = n$. With this in mind, it is easy to see that $g(f(n)) = h(f(n)) = n$ for all integers n.

(9) Construct two right inverses for $f : \mathbf{R} \longrightarrow \mathbf{R}^+ \cup \{0\}$ defined by $f(x) = |x|$.

Solution. The functions $g(x) = x$ and $h(x) = -x$ both serve as right inverses for f (only because the domain of these functions is the set of nonnegative real numbers). Indeed, we see that $f(g(x)) = |x| = x$ and $f(h(x)) = |-x| = x$.

(14) Construct two left inverses for the function $f : X \longrightarrow Y$ given below. Assume that $Y = \{1,2,3,4,5,6\}$.

$$f : \begin{pmatrix} a & b & c & d \\ 1 & 3 & 2 & 5 \end{pmatrix}$$

Solution. In constructing the left inverses, we can assign $4, 6$ to any member of X. Consequently, there are actually twelve left inverses for this function. Here are two examples.

$$g : \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & d & b & b & d & b \end{pmatrix} \qquad h : \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & d & b & c & d & c \end{pmatrix}$$

(15) Construct two right inverses for the function $f : X \longrightarrow Y$ given below. Assume that $Y = \{1,2,3,4,5,6\}$.

$$f : \begin{pmatrix} a & b & c & d & e & f & g & h \\ 6 & 2 & 3 & 2 & 4 & 4 & 5 & 1 \end{pmatrix}$$

Solution. Since $\mathtt{Pre}_f(4) = \{e, f\}$, there are two right inverses we can construct, namely

$$g : \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ h & b & c & e & g & a \end{pmatrix} \qquad h : \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ h & b & c & f & g & a \end{pmatrix}$$