MATH 4510 REVIEW FOR EXAM I

In Problems 1-4, let $X = \{a, b, c, d, e\}$.

1. Determine which of the following binary relations on the set X are reflexive, symmetric, or transitive. For those that are not, give a specific counterexample to show why.
   (a) $\theta = \{(a, a), (e, e)\}$
   (b) $\alpha = \{(b, d), (d, e), (a, a), (d, b), (b, b), (e, d)\}$
   (c) $\beta = \{(a, a), (a, b), (b, b), (c, c), (c, a), (e, e), (a, c), (d, d), (c, b)\}$
2. Construct the partition on X induced by the equivalence relation θ given below.
   $$\theta = \{(b, e), (a, a), (e, a), (b, c), (b, b), (c, b), (e, c), (a, e), (c, c), (d, d), (a, c), (e, e), (a, b), (b, a), (c, e), (c, a), (e, b)\}$$
3. Construct the equivalence relation on X induced by the partition
   $$\mathcal{F} = \{\{a, c, e\}, \{b, d\}\}$$
4. Consider the family of subsets $\mathcal{F} = \{\{a, c, e\}, \{b, c, d\}\}$. Construct the binary relation $\alpha = \{(x, y) \in X \times X : x, y \in S \text{ for some } S \in \mathcal{F}\}$. Is this an equivalence relation?
5. Determine whether the following binary relations are reflexive, symmetric, or transitive. If they are not, give a specific counterexample to indicate which property or properties fail.
   (a) $\theta \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $(m, n) \in \theta$ if and only if $m - n$ is even.
   (b) $\alpha \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+$ defined by $(m, n) \in \alpha$ if and only if $|m - n| \leq 1$.
   (c) $\beta \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+$ defined by $(m, n) \in \beta$ if and only if mn is odd.
6. Let $U = \{1, 2, 3\}$ and consider the binary relation λ on $U^2 \times U$ defined by
   $$\lambda = \{([1, 1], 1), ([1, 2], 3), ([1, 3], 3), ([2, 1], 3), ([2, 2], 1), ([2, 3], 3), ([3, 1], 3), ([3, 2], 3), ([3, 3], 1)\}$$
   (a) Explain why this relation is a binary operation on U.
   (b) Is this a commutative operation on U?
   (c) Is this an associative operation on U?
7. Consider the binary operation $\circledcirc : \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}$ defined by $m \circledcirc n = |2m - n|$.
   (a) Is this operation commutative?
   (b) Is this operation associative?
8. Consider the binary operation $\dagger : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ defined by $x \dagger y = \sqrt[4]{x^4 + y^4}$.
   (a) Is this operation commutative?
   (b) Is this operation associative?
9. A binary operation on the set $X = \{a, b, c\}$ has been partially defined by the table below. If we know that the operation is commutative, complete the table.

$*$	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$
$\boldsymbol{a}$	c	a	
$\boldsymbol{b}$		b	c
$\boldsymbol{c}$	a		b

10. What is the remainder obtained when 6^{100} is divided by 40?

11. Which of the following integers is a member of $[2241]_{25}$?

$$a = 66 \qquad b = -59 \qquad c = 118$$

12. What is the smallest positive representative of $[3141]_{70} \boxplus_{70} [-1414]_{70}$?

13. Write out the operation table for $\mathbb{Z}_6$ under multiplication modulo 6.

14. Compute $f \circ g$, $f \circ g^{-1}$, and $g \circ f^{-1}$ for the permutations given below.

$$f : \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \qquad g : \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$$