MATH 1910 QUIZ 3

20 points

NAME:____________________________

10 pts 1. Use the Chain Rule to differentiate the function $g(x) = \left(x^2 + \sin(x)\right)^{-4}$. You must show your steps for full credit.

Solution. Let $u(x) = x^2 + \sin(x)$ and let $f(u) = u^{-4}$. We know

$$\begin{aligned}
\frac{dg}{dx} &= \frac{df}{du} \cdot \frac{du}{dx} \\
&= \frac{d}{du}\left[u^{-4}\right] \cdot \frac{d}{dx}\left[x^2 + \sin(x)\right] \\
&= \frac{d}{du}\left[u^{-4}\right] \cdot \left(\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[\sin(x)\right]\right) \\
&= -4u^{-5} \cdot (2x + \cos(x)) \\
&= -\frac{8x + 4\cos(x)}{\left(x^2 + \sin(x)\right)^5}
\end{aligned}$$

10 pts 2. Find a formula for $\dfrac{dy}{dx}$ if $\cos(y) = \tan(x)$. You must show your steps for full credit.

Solution. We are differentiating with respect to x and will therefore need the chain rule to differentiate $\cos(y)$. In this case, let $u(y) = y$ and let $f(u) = \cos(u)$. We know

$$\begin{aligned}
\cos(y) = \tan(x) \quad &\Longrightarrow \quad \frac{d}{dx}\left[\cos(y)\right] = \frac{d}{dx}\left[\tan(x)\right] \\
&\Longrightarrow \quad \frac{df}{du} \cdot \frac{du}{dx} = \frac{d}{dx}\left[\tan(x)\right] \\
&\Longrightarrow \quad \frac{d}{du}\left[\cos(u)\right] \cdot \frac{du}{dx} = \frac{d}{dx}\left[\tan(x)\right] \\
&\Longrightarrow \quad -\sin(u)\frac{du}{dx} = \sec^2(x) \\
&\Longrightarrow \quad -\sin(y)\frac{dy}{dx} = \sec^2(x) \\
&\Longrightarrow \quad \frac{dy}{dx} = -\frac{\sec^2(x)}{\sin(y)}
\end{aligned}$$