MATH 1910 QUIZ 4

20 points

NAME:__________________________

In the problems that follow, let $f(x) = \frac{1}{6}(2x^3 - 3x^2 - 12x) + 1$.

10 pts 1. Compute $f'(x)$ and use the derivative to identify the critical numbers for f. Do not classify the critical numbers. You must show your work for full credit.

Solution. First, observe that $f'(x) = x^2 - x - 2 = (x+1)(x-2)$. The critical numbers for f will occur when $f'(x) = 0$. Now,

$$f'(x) = 0 \Longrightarrow (x+1)(x-2) = 0 \Longrightarrow x = -1 \text{ or } x = 2$$

Therefore, f has two critical points, namely $x = -1$ and $x = 2$.

10 pts 2. Use the First Derivative Test to determine whether the critical numbers for f produce relative maximum or minimum outputs for f. You must show your work for full credit.

Solution. To apply the First Derivative Test, we need to select a "test number" from each of the three subsets of the real number line determined by the critical numbers.

- Select $x = -2$ from the ray $(-\infty, -1]$. Since $f'(-2) = 8 > 0$, we know the graph of f in INCREASING to the left of $x = -1$.
- Select $x = 0$ from the interval $[-1, 2]$. Since $f'(0) = -2 < 0$, we know the graph of f is DECREASING between $x = -1$ and $x = 2$.
- Select $x = 3$ from the ray $[2, +\infty)$. Since $f'(3) = 4 > 0$, we know the graph of f is INCREASING to the right of $x = 2$.

Since the graph of f changes from increasing to decreasing as we cross over $x = -1$, we know that f has a relative maximum output at $x = -1$. Since the graph of f changes from decreasing to increasing as we cross over $x = 2$, we know that f has a relative minimum output at $x = 2$.