MATH 1910 EXAM I

100 points

NAME:________________________

Please place the letter of your selection in the blank provided. These questions are worth five points each.

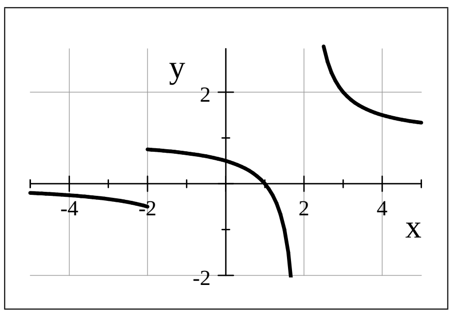
5 pts 1. **D** The function $f(x) = \dfrac{x}{1-x^2}$ is

(a) continuous everywhere.
(b) discontinuous only at $x = 0$.
(c) discontinuous only at $x = 1$.
(d) discontinuous only at $x = 1$ and $x = -1$.
(e) discontinuous only at $x = 0$, $x = 1$ and $x = -1$.

Justification: Note that $1 - x^2 = 0$ when $x = \pm 1$. These values of x result in division by 0 for the function f and hence are points of discontinuity. The function f actually has vertical asymptotes at both of these input values.

Problems 2-4 refer to the function whose formula is given below.

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{if} \quad x \le -2 \\[2ex] \dfrac{x-1}{x-2} & \text{if} \quad -2 < x \end{cases}$$



5 pts 2. **C** The function f has a vertical asymptote

(a) at no value of x.
(b) only at $x = 0$.
(c) only at $x = 2$.
(d) only at $x = 0$ and $x = 2$.
(e) only at $x = 1$.

5 pts 3. **D** The function f has a discontinuity

(a) at no value of x.
(b) only at $x = -2$.
(c) only at $x = 0$.
(d) only at $x = 2$ and $x = -2$.
(e) only at $x = 0$, $x = 2$, and $x = -2$. .

5 pts 4. **A** Which of the following statements is true at $x = 1$?

(a) f is continuous.
(b) f has a jump discontinuity.
(c) f has a removable discontinuity.
(d) f has a vertical asymptote.
(e) f is undefined.

Justification: The graph of f has no jump, tear, or hole at the input value $x = 1$.

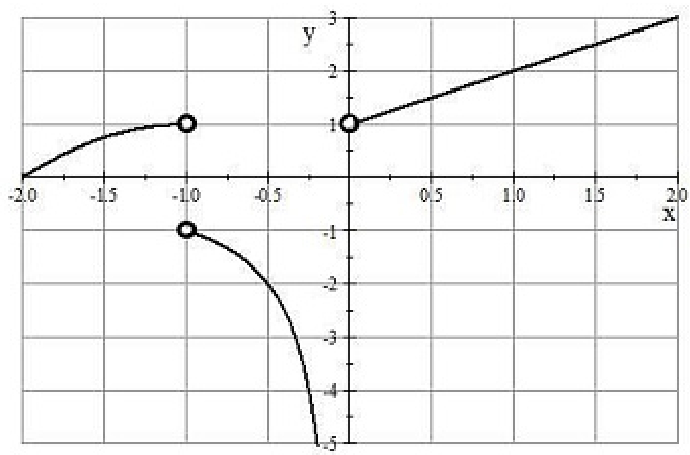
5 pts 5. **D** Which of the following limits would be used to determine the formula for the derivative function of $g(x) = 3x^2 - 1$?

(a) $\lim_{h \longrightarrow 0} \left(3h^2 - 1\right)$ (b) $\lim_{h \longrightarrow 0} \left(3h^2 + 6h + 3h^2 - 1\right)$

(c) $\lim_{h \longrightarrow 0} \frac{3(x+h)^2 - 1}{h}$ (d) $\lim_{h \longrightarrow 0} \frac{\left[3(x+h)^2 - 1\right] - \left[3x^2 - 1\right]}{h}$

(e) $\lim_{h \longrightarrow 0} \frac{\left[3(x+h)^2\right] - 3x^2}{h}$

Problems 6-8 refer to the function f whose graph is given below.



5 pts 6. **E** According to the graph above, we have

(a) $\lim_{x \longrightarrow 0^+} f(x) = -\infty$ (b) $\lim_{x \longrightarrow 0^+} f(x) = .5$

(c) $\lim_{x \longrightarrow 0} f(x) = .5$ (d) $\lim_{x \longrightarrow 0} f(x) = -5$

(e) $\lim_{x \longrightarrow 0^+} f(x) = 1$

5 pts 7. **B** According to the graph above, we have

(a) $\lim_{x \longrightarrow -1^-} f(x) = 0$ (b) $\lim_{x \longrightarrow -1^+} f(x) = -1$

(c) $\lim_{x \longrightarrow -1^-} f(x) = -1$ (d) $\lim_{x \longrightarrow -1} f(x) = 0$

(e) $\lim_{x \longrightarrow -1} f(x) = -1$

5 pts 8. **C** According to the graph above, the function f has a non-removable discontinuity

(a) at no value of x. (b) only at $x = 0$.

(c) only at $x = -1$ and $x = 0$. (d) only at $x = -1$.

(e) only at $x = 1$.

Justification: The graph of f has a jump at $x = -1$ and a tear at $x = 0$. These input values therefore represent non-removable discontinuities for f. The function does not have jumps, holes, or tears at any other value of x.

5 pts 9. ____**A**________Suppose the tangent line to a function f at the point $(3, f(3))$ is given by $y = 4.5x - 3.8$. What is the value of $f'(3)$?

(a) $f'(3) = 4.5$ (b) $f'(3) = -3.8$
(c) $f'(3) = 3$ (d) $f'(3) = -9.7$
(e) $f'(3) = 0$

15 pts 10. Consider the function $f(x) = \dfrac{x-3}{x^2-3x}$.

(a) At what values of x will f have discontinuities? Show your work. (Do not classify the discontinuities yet.)

Solution. Discontinuities will occur at input values that produce division by 0. Observe

$$x^2 - 3x = 0 \Longrightarrow x(x-3) = 0 \Longrightarrow x = 0 \quad \text{or} \quad x = 3$$

(b) Are any of these discontinuities non-removable? Justify your answer.

Solution. Observe that

$$f(x) = \frac{x-3}{x(x-3)} = \frac{1}{x} \text{ as long as } x \neq 3$$

Based on this simplification, we see that $x = 0$ is a non-removable discontinuity. (Also, $x = 3$ is a removable discontinuity.)

(c) Are any of these discontinuities removable? If so, determine the limit of $f(x)$ as x approaches this x-value. You must show your steps for full credit.

Solution. As mentioned in Part (b), $x = 3$ serves as a removable discontinuity for f. Now

$$\lim_{x \longrightarrow 3} f(x) = \lim_{x \longrightarrow 3} \frac{1}{x} = \frac{1}{3}$$

10 pts 11. Consider the function $f(x) = \dfrac{4x^4 - 3x + 1}{5x^4 - 8}$. Evaluate $\displaystyle\lim_{x \longrightarrow -\infty} f(x)$. You must show your steps for full credit.

Solution. Observe

$$\lim_{x \longrightarrow -\infty} \frac{4x^4 - 3x + 1}{5x^4 - 8} = \lim_{x \longrightarrow -\infty} \frac{4x^4}{5x^4} = \frac{4}{5}$$

10 pts 12. Use the limit definition of the derivative to determine the formula for the derivative function of $g(x) = 3x^2 - 1$. You must show all of your steps for full credit.

Solution. Observe

$$\begin{aligned}
g'(x) &= \lim_{h \longrightarrow 0} \frac{g(x+h) - g(x)}{h} \\
&= \lim_{h \longrightarrow 0} \frac{\left[3(x+h)^2 - 1\right] - \left[3x^2 - 1\right]}{h} \\
&= \lim_{h \longrightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 + 1}{h} \\
&= \lim_{h \longrightarrow 0} \frac{6xh + 3h^3}{h} \\
&= \lim_{h \longrightarrow 0} (6x + 3h) \\
&= 6x
\end{aligned}$$

10 pts 13. The derivative function for $g(x) = 3x^2 - 1$ is given by $g'(x) = 6x$. What is the formula for the tangent line to the graph of g at the point $(-2, g(-2))$? Show your steps.

Solution. We know that $g(-2) = 11$ and we know $g'(-2) = -12$. Therefore, the tangent line will be given by the formula

$$y - 11 = -12(x + 2) \quad \text{or} \quad y = -12x - 13$$

10 pts 14. The graph of a function f is shown below. On the grid provided, draw a rough sketch of its derivative function f'.