MATH 1910 EXAM II

100 points

NAME:____________________________

Write down the derivative function for each function below. These are worth five points each.

(1) $f(x) = \cos(x)$ $\qquad$ $f'(x) = -\sin(x)$

(2) $g(y) = \ln(y)$ $\qquad$ $g'(y) = \dfrac{1}{y}$

(3) $h(z) = z^{4/3}$ $\qquad$ $h'(z) = \dfrac{4}{3}z^{1/3}$

(4) $k(a) = 5^a$ $\qquad$ $k'(a) = 5^a \ln(5)$

(5) $j(b) = b^{-4}$ $\qquad$ $j'(b) = -4b^{-5}$

Suppose f and g are differentiable functions. Match each derivative on the right with the general rule used to compute it. These are worth five points each.

(6) **B** $\left(\dfrac{f}{g}\right)' =$ $\qquad$ (A) $f' + g'$

(7) **A** $(f+g)' =$ $\qquad$ (B) $\dfrac{gf' - fg'}{g^2}$

(8) **D** $(fg)' =$ $\qquad$ (C) $gf' - fg'$

(D) $gf' + fg'$

(E) $f'(g)g'$

5 pts **9.** **D** If we want to use the Chain Rule to differentiate $g(x) = \tan\left(x^{4/3}\right)$, we know $\frac{dg}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$, where

(a) $f(u) = x$ and $u(v) = \tan^{4/3}(x)$ $\qquad$ (b) $u(x) = \tan(x)$ and $f(x) = x^{4/3}$

(c) $u(x) = x$ and $f(x) = \tan^{4/3}(x)$ $\qquad$ (d) $u(x) = x^{4/3}$ and $f(u) = \tan(u)$

(e) $u(x) = x$ and $f(x) = \tan(x^{1/3})$

5 pts **10.** **E** If we want to use the Product Rule to differentiate $g(x) = x\ln(x)$, then we know

(a) $\frac{dg}{dx} = x \cdot \frac{d}{dx}[\ln(x)]$

(b) $\frac{dg}{dx} = \ln(x) \cdot \frac{d}{dx}[x]$

(c) $\frac{dg}{dx} = \frac{d}{dx}[x] \cdot \frac{d}{dx}[\ln(x)]$

(d) $\frac{dg}{dx} = x \cdot \frac{d}{dx}[\ln(x)] - \ln(x) \cdot \frac{d}{dx}[x]$

(e) $\frac{dg}{dx} = x \cdot \frac{d}{dx}[\ln(x)] + \ln(x) \cdot \frac{d}{dx}[x]$

10 pts **11.** Find the derivative of $f(x) = x\ln(x) - x$. You must show your work for full credit.

Solution. We know that

$$\begin{aligned} f'(x) &= \frac{d}{dx}[x\ln(x) - x] \\ &= \frac{d}{dx}[x\ln(x)] - \frac{d}{dx}[x] \\ &= \frac{d}{dx}[x]\ln(x) + x\frac{d}{dx}[\ln(x)] - \frac{d}{dx}[x] \\ &= (1)\ln(x) + x\left(\frac{1}{x}\right) - 1 \\ &= \ln(x) + 1 - 1 \\ &= \ln(x) \end{aligned}$$

10 pts **12.** Find the derivative of $g(x) = \tan(x^{4/3})$. You must show your work for full credit.

Solution. Let $u(x) = x^{4/3}$ and let $f(u) = \tan(u)$. Then

$$\begin{aligned} g'(x) &= \frac{df}{du}\frac{du}{dx} \\ &= \frac{d}{du}[\tan(u)]\frac{d}{dx}\left[x^{4/3}\right] \\ &= \sec^2(u)\left(\frac{4}{3}x^{1/3}\right) \\ &= \frac{4x^{1/3}\sec^2(x^{4/3})}{3} \end{aligned}$$

10 pts **13.** Find the derivative of $f(x) = \dfrac{3x}{2-x}$. You must show your work for full credit.

Solution. We know that

$$\begin{aligned} f'(x) &= \left[\frac{1}{(2-x)^2}\right]\left[(2-x)\frac{d}{dx}[3x] - 3x\frac{d}{dx}[2-x]\right] \\ &= \frac{(2-x)(3) - (3x)(-1)}{(2-x)^2} \\ &= \frac{6 - 3x + 3x}{(2-x)^2} \\ &= \frac{6}{(2-x)^2} \end{aligned}$$

10 pts **14.** Find the equation of the tangent line to the graph of $f(x) = \dfrac{3x}{2-x}$ at the point $(1, f(1))$. You may use Problem 13. You must show your work for full credit.

Solution. We know that the equation of the tangent line will be $T(x) = f'(1)[x-1] + f(1)$. Now, $f(1) = 3$ and $f'(1) = 6$. Therefore, the formula for the tangent line is

$$T(x) = 6(x-1) + 3 \qquad \text{OR} \qquad T(x) = 6x - 3$$

10 pts **15.** Find a formula for $\dfrac{dy}{dx}$ if $y^3 - y = x^3$. You must show your work for full credit.

Solution. Let $u(y) = y$ and let $f(u) = u^3$. Observe

$$\begin{aligned}
y^3 - y = x^3 \quad &\implies \quad \frac{d}{dx}\left[y^3 - y\right] = \frac{d}{dx}\left[x^3\right] \\
&\implies \quad \frac{d}{dx}\left[y^3\right] - \frac{d}{dx}\left[y\right] = \frac{d}{dx}\left[x^3\right] \\
&\implies \quad \frac{df}{du}\frac{du}{dx} - \frac{du}{dx} = \frac{d}{dx}\left[x^3\right] \\
&\implies \quad \frac{d}{du}\left[u^3\right]\frac{du}{dx} - \frac{du}{dx} = \frac{d}{dx}\left[x^3\right] \\
&\implies \quad 3u^2\frac{du}{dx} - \frac{du}{dx} = 3x^2 \\
&\implies \quad 3y^2\frac{dy}{dx} - \frac{dy}{dx} = 3x^2 \\
&\implies \quad \left(3y^2 - 1\right)\frac{dy}{dx} = 3x^2 \\
&\implies \quad \frac{dy}{dx} = \frac{3x^2}{3y^2 - 1}
\end{aligned}$$

10 pts **BONUS:** The radius R of a circular disk is measured to be 24 inches, with a tolerance of ± 0.1 inches. Use the differentials to estimate the maximum error in the computed area of the disk.

Solution. The area A of a circular disk (measured in square feet) can be written as a function of the radius R (measured in feet) according to the rule

$$A = f(R) = \pi R^2$$

Therefore, the differential for A is $dA = 2\pi R \cdot dR$. The maximum error in the computed area of the disk can therefore be estimated as

$$dA = 2\pi(24) \cdot (0.1) \approx 15.08 \text{ square inches}$$