MATH 1910 Practice for EXAM I

Please place the letter of your selection in the blank provided. These questions are worth five points each.

5 pts 1. ___**C**___ The function f below has a discontinuity when $b = 0$. Which of the functions shown is equal to the function f when $b \neq 0$?

$$f(b) = \left(\frac{1}{b}\right)\left(\frac{1}{b^2-1} + \frac{1}{b^2+1}\right)$$

(a) $g(b) = \dfrac{1}{b-1} + \dfrac{1}{b^2+1}$ (b) $g(b) = \dfrac{1}{b-1} + \dfrac{1}{b+1}$

(c) $g(b) = \dfrac{2b}{(b^2-1)(b^2+1)}$ (d) $g(b) = \dfrac{2}{(b^2-1)}$

(e) $g(b) = \dfrac{1}{b-1}$

$$\begin{aligned} f(b) &= \left(\frac{1}{b}\right)\left(\frac{1}{b^2-1} + \frac{1}{b^2+1}\right) \\ &= \left(\frac{1}{b}\right)\left[\left(\frac{1}{b^2-1}\right)\left(\frac{b^2+1}{b^2+1}\right) + \left(\frac{1}{b^2+1}\right)\left(\frac{b^2-1}{b^2-1}\right)\right] \\ &= \left(\frac{1}{b}\right)\left[\frac{2b^2}{(b^2-1)(b^2+1)}\right] \\ &= \frac{2b}{(b^2-1)(b^2+1)} \qquad (b \neq 0) \end{aligned}$$

5 pts 2. ___**A**___ The function $f(x) = \dfrac{x^2-1}{x^2+2x+1}$ has a removable discontinuity

(a) at no value of x. (b) only at $x = 1$.
(c) only at $x = 0$, $x = 1$, and $x = -1$ (d) only at $x = -1$ and $x = 1$.
(e) only at $x = -1$.

$$f(x) = \frac{x^2-1}{x^2+2x+1} = \frac{(x-1)(x+1)}{(x+1)(x+1)} = \frac{x-1}{x+1}$$

The function f has a discontinuity at $x = -1$. However, this discontinuity is a vertical asymptote for the function f.

Problems 3-5 refer to the function whose formula is given below.

$$f(x) = \begin{cases} \dfrac{3}{x^2} & \text{if } \ x \leq -2 \\[2ex] \dfrac{x-1}{x-2} & \text{if } \ -2 < x \end{cases}$$

5 pts 3. ___**C**___ Does the function f have a vertical asymptote?

(a) No. (b) Yes, only at $x = 0$.
(c) Yes, only at $x = 2$. (d) Yes, only at $x = 0$ and $x = 2$.
(e) Yes, only at $x = 1$.

Notice the the subformula $g(x) = 3x^{-2}$ has a vertical asymptote at $x = 0$; however, this subformula is not valid on the input interval that contains $x = 0$. On the other hand, the subformula $h(x) = (x-1)/(x-2)$ has a vertical asymptote at $x = 2$. Since this subformula is valid on the input interval that contains $x = 2$, we may conclude that f has a vertical asymptote.

5 pts 4. **A** Does the function f have any jump discontinuities?

(a) No, the function is continuous. (b) Yes, only at $x = -2$.
(c) Yes, only at $x = 0$. (d) Yes, only at $x = 2$ and $x = -2$.
(e) Yes, only at $x = 0$, $x = 2$, and $x = -2$. .

Aside from the vertical asymptote at $x = 2$, the function will be continuous except *possibly* at the branch point $x = -2$. First, note that $f(-2) = 3/4$. Second, observe that

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{3}{x^2} = \frac{3}{4} \qquad \lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} \frac{x-1}{x-2} = \frac{3}{4}$$

Consequently, we may conclude that the function f is continuous at the branch point.

5 pts 5. **B** Which of the following statements is true at $x = -2$?

(a) f is undefined. (b) $f(-2) = 3/4$.
(c) f has a removable discontinuity. (d) f has a vertical asymptote.
(e) $f(-2) = -2$.

5 pts 6. **C** Consider the function f and the three statements below.

$$v = f(y) = \frac{1 + 3\sqrt{y}}{y - 2}$$

I. We know that $f(y) = \dfrac{3}{\sqrt{y}}$.

II. We know that $\lim_{y\to+\infty} f(y) = \lim_{y\to+\infty} \dfrac{3}{\sqrt{y}}$.

III. We know that the function f has a horizontal asymptote at the line $v = 0$.

(a) All three statements are true. (b) Only Statements I and II are true.

(c) Only Statements II and III are true. (d) Only Statements I and III are true.

(e) None of these statements is true.

Observe that the function f is undefined when $x = 2$, while the formula $3\sqrt{y}/\sqrt{y}$ is defined when $x = 2$. Consequently, Statement I must be false. On the other hand, we know that *in the limiting process* as the values of x grow more and more positive, we have

$$\lim_{y\to+\infty} f(y) = \lim_{y\to+\infty} \frac{3}{\sqrt{y}} = 0$$

5 pts 7. **E** What is the average rate of change for the function $y = f(x) = \dfrac{2 + 3x^{3/2}}{x}$ on the input interval $1 \le x \le 9$?

(a) Approximately 5.00 (b) Approximately 1.153

(c) Approximately 9.22 (d) Approximately 4.22

(e) Approximately 0.528

$$\frac{f(9)-f(1)}{9-1} = \left(\frac{1}{8}\right)\left[\frac{83}{9} - 5\right] \approx 0.528$$

5 pts 8. ____**C**________Sarah started walking from her house to a store three miles away at 3:00 PM. Which of the following is a properly defined *varying* quantity in this situation?

(a) the distance from Sarah's house to the store, measured in feet

(b) the time

(c) the distance in feet that Sarah has walked from her house since 3:00 PM

(d) the total distance in miles that Sarah has walked when she arrives at the store

(e) Sarah

Defining a *quantity* requires us to identify the quantity, describe the units used to measure the quantity; and when possible, state the reference point for the measurement. Items (b) and (c) fail to meet these criteria and therefore do not defined quantities. Items (a) and (d) define *constant* quantities in this problem. Only Item (c) defines a varying quantity.

Problems 9 and 10 refer to the following scenario. Toby is traveling toward Knoxville on Interstate 40. Let d represent the number of miles Toby has driven since taking the on-ramp, and let t represent the number of minutes passed since Toby passed a rest-stop located two miles from the on-ramp. Once past the rest-stop, for every increase of five miles since Toby got on the on-ramp, the time since Toby passed the rest-stop increases by three minutes.

5 pts 9. ____**D**________ Which of the following formulas is correct?

(a) $d = \frac{5}{3}\Delta t$

(b) $\Delta t = \frac{5}{3}\Delta d$

(c) $t = \frac{3}{5}d$

(d) $\Delta t = \frac{3}{5}\Delta d$

(e) More than one of these statements is true.

We are told that whenever $\triangle d = 5$ miles, then $\triangle t = 3$ minutes. Consequently, we know that

$$\frac{\triangle d}{\triangle t} = \frac{5}{3} \qquad \text{or} \qquad 3\triangle d = 5\triangle t$$

Now, when $t = 0$ minutes, we know that $d = 2$ miles; consequently, Item (c) is incorrect.

5 pts 10. ____**B**________ Eight minutes after passing the rest-stop, Tody is eleven miles from the on-ramp. How many minutes have elapsed since Toby passed the rest-stop when he is six miles from the on-ramp?

(a) 11 minutes

(b) 5 minutes

(c) $3\frac{1}{3}$ minutes

(d) $16\frac{1}{3}$ minutes

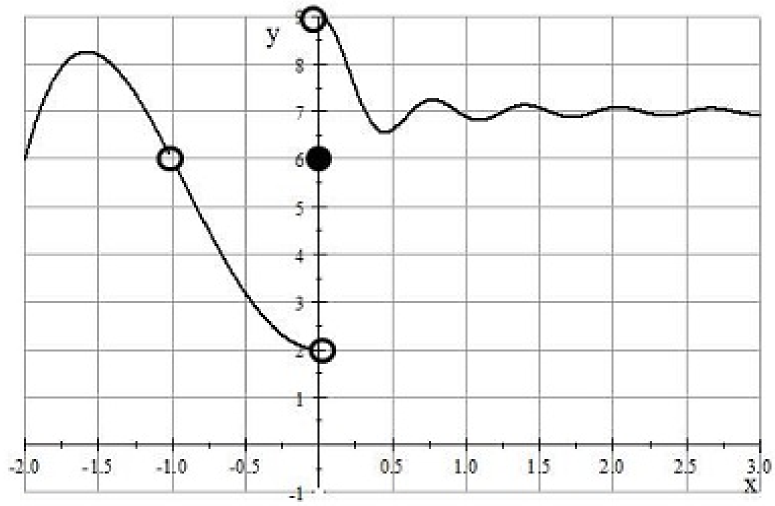
(e) $-\frac{1}{3}$ minute

When $d = 11$ miles, we know that $t = 8$ minutes. Decreasing the distance from the on-ramp to $d = 6$ miles gives us $\triangle d = -5$ miles. Therefore, the corresponding change in time will be

$$\triangle t = \frac{3}{5}(-5) = -3$$

Therefore, we know that $t = 8 - 3 = 5$ minutes.

Problems 11-13 refer to the function $y = f(x)$ whose graph is given below.



5 pts 11. **B** According to the graph above, we have

(a) $\lim_{x \longrightarrow 0^+} f(x) = 2$ (b) $\lim_{x \longrightarrow 0^+} f(x) = 9$
(c) $\lim_{x \longrightarrow 0^+} f(x) = 6$ (d) $\lim_{x \longrightarrow 0^+} f(x) = 5$
(e) $\lim_{x \longrightarrow 0^+} f(x)$ does not exist

5 pts 12. **C** According to the graph above, we have

(a) $\lim_{x \longrightarrow --1} f(x)$ does not exist (b) $\lim_{x \longrightarrow -1} f(x) = 1$
(c) $\lim_{x \longrightarrow -1} f(x) = 6$ (d) $\lim_{x \longrightarrow -1} f(x) = 0$
(e) $\lim_{x \longrightarrow -1} f(x) = 0$

5 pts 13. **E** According to the graph above, we know that

(a) $\lim_{x \longrightarrow +\infty} f(x)$ does not exist (b) $\lim_{x \longrightarrow +\infty} f(x) = 0$

(c) $\lim_{x \longrightarrow +\infty} f(x) = -\infty$ (d) $\lim_{x \longrightarrow +\infty} f(x) = +\infty$

(e) $\lim_{x \longrightarrow +\infty} f(x) = 7$

5 pts 14. **A** If $f(x) = \dfrac{1}{x-1}$, which of the following functions gives the average rate of change for f from $x = a$ to $x = a + h$ for nonzero values of h?

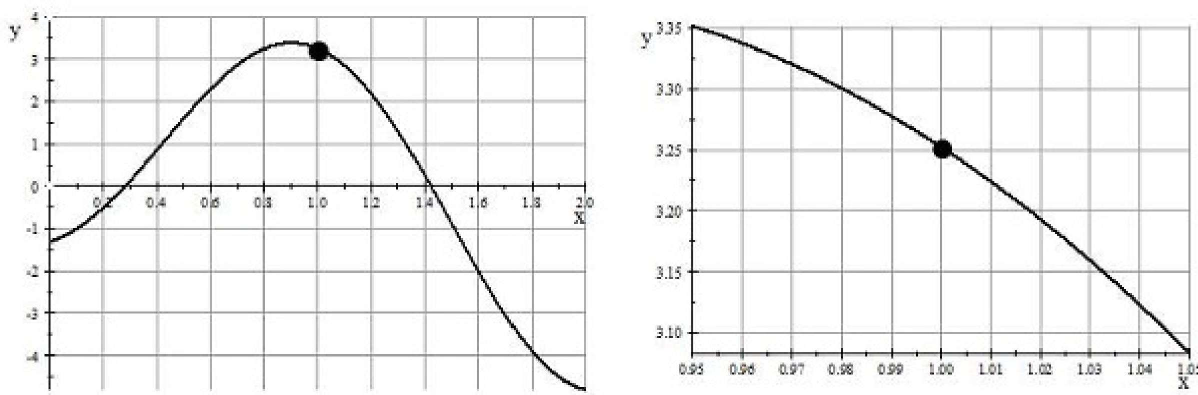
(a) $g_a(h) = -\dfrac{1}{(a-1)(a-1+h)}$ (b) $g_a(h) = -\dfrac{h}{(a-1)^2}$

(c) $g_a(h) = \dfrac{1}{a^2-1+h}$ (d) $g_a(h) = \dfrac{1}{h(a^2-1)}$

(e) $g_a(h) = -\dfrac{h}{(a-1+h)^2}$

$$
\begin{aligned}
g_a(h) &= \frac{f(a+h)-f(a)}{h} \\
&= \left(\frac{1}{h}\right)\left[\frac{1}{a+h-1} - \frac{1}{a-1}\right] \\
&= \left(\frac{1}{h}\right)\left[\frac{(a-1)-(a+h-1)}{(a-1)(a-1+h)}\right] \\
&= \left(\frac{1}{h}\right)\left[-\frac{h}{(a-1)(a-1+h)}\right] \\
&= -\frac{1}{(a-1)(a-1+h)} \qquad (h \neq 0)
\end{aligned}
$$

5 pts 15. ____**C**____ The diagram below shows the graph of a function f along with a zoom-in on the point $(1.00, 3.252)$. What is the approximate local constant rate of change for $f(x)$ with respect to x near the input value $x = 1.00$?



(a) Approximately 3.00 (b) Approximately -1.25

(c) Approximately -2.40 (d) Approximately 2.70

(e) Approximately -3.00

Individual answers may vary; however, the most efficient way to estimate the locat constant rate of change is to compute the average rate of change for the function f over a small input interval that begins or ends at $x = 1$. For example, if we consider the input interval $0.98 \leq x \leq 1$, we know that the average rate of change will be

$$\frac{f(0.98) - f(1)}{0.98 - 1} \approx -\frac{3.30 - 3.252}{0.02} = -2.40$$

5 pts 16. ____**D**____ If $f(x) = x + \sqrt{x}$, then the function g_4 that gives the average rate of change for f on the interval from $x = 4$ to $x = 4 + h$ is given by

$$g_4(h) = 1 + \frac{\sqrt{4+h} - 2}{h}$$

What would be the approximate local constant rate of change of $f(x)$ with respect to x near $x = 4$?

(a) Approximately 3.00 (b) Approximately 0.24

(c) Approximately 2.12 (d) Approximately 1.25

(e) Approximately 0.33

As in the previous problem, the best way to estimate the local constant rate of change for the function f is to compute the average rate of change for f over a small input interval that begins or ends at $x = 4$. For example, if we consider the input interval $4 \le x \le 4.01$, then $h = 0.01$, and we know that

$$g_4(0.01) = 1 + \frac{\sqrt{4.01} - 2}{0.01} \approx 1.25$$

5 pts 17. **B** Let $y = f(x)$ be a function that is locally linear at the point $(-1, f(-1))$, and suppose that $y = -2.27(x+1) - 4.18$ is the formula for the tangent line to the graph of f at the point $(-1, f(-1))$. Which of the following statements would be true about the *average rate of change* for the function f on the input interval $-1.01 \le x \le -1$?

(a) It would be approximately 2.27. (b) It would be approximately -2.27.

(c) It would be approximately 4.18. (d) It would be approximately -4.18.

(e) It would be approximately 2.27/4.18.

If the tangent line to the graph of f exists at the point $(-1, f(-1))$, then the slope of the tangent line can be approximated by computing the average rate of change for the function f on small input intervals that begin or end at $x = -1$. Conversely, the slope of the tangent line will be an approximation to these average rates of change. The approximation improves the smaller the input interval we consider.

5 pts 18. **E** What can be said about the limiting process $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1+9x^2}}{4x-2}$?

(a) $\lim\limits_{x \longrightarrow --\infty} \dfrac{\sqrt{1+9x^2}}{4x-2} = 1$ (b) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1+9x^2}}{4x-2} = -1$

(c) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1+9x^2}}{4x-2} = \dfrac{3}{4}$ (d) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1+9x^2}}{4x-2} = 0$

(e) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1+9x^2}}{4x-2} = -\dfrac{3}{4}$

$$\lim_{x \to -\infty} \frac{\sqrt{1+9x^2}}{4x-2} = \lim_{x \to -\infty} \frac{\sqrt{9x^2}}{4x} = \lim_{x \to -\infty} \frac{3\,|x|}{4x} = -\frac{3}{4}$$

5 pts 19. **B** Suppose that a and b are distinct real numbers. Which of the following statements is true about the function whose formula is given below?

$$f(t) = \frac{(t-a)^2(t-b)}{(t-a)(t-b)^2}$$

(a) The function f has a removable discontinuity at $x = a$ and at $x = b$.

(b) The function f has a removable discontinuity at $x = a$ and a vertical asymptote at $x = b$.

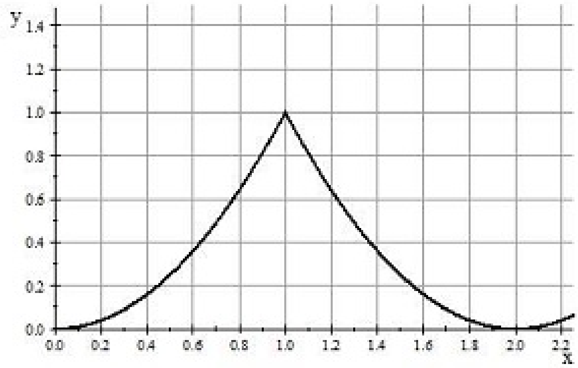
(c) The function f has a vertical asymptote at $x = a$ and a removable discontinuity at $x = b$.

(d) The function f has vertical asymptote at $x = a$ and at $x = b$.

(e) None of the above statements is true.

$$f(t) = \frac{(t-a)^2(t-b)}{(t-a)(t-b)^2} = \frac{t-a}{t-b} \qquad (a \ne 0)$$

5 pts 20. ____**D**____ Consider the function $y = f(x)$ whose graph is shown along with the statements below the graph.



I. The function f is continuous at the input value $x = 1$.

II. The function f is locally linear at the point $(1, 1)$.

III. The function f is *not* locally linear at the point $(1, 1)$.

(a) Only Statement I is true.
(b) Only Statement II is true.
(c) Only Statement III is true.
(d) Statements I and II are true.
(e) Statements I and III are true.

First, notice that the sharp"kink" in the graph of the function f means that, no matter how much we zoom-in on the graph at the point $(1, 1)$, the graph will *never* look like a straight line near this point. (In fact, the graph will always look the same.) Consequently, Statement II is false. However, observe that

$$\lim_{x\to 1^-} f(x) = 1 = f(1) \qquad \lim_{x\to 1^+} f(x) = 1 = f(1)$$

Therefore, the function f is continuous at $x = 1$.