MATH 1910 EXAM I

100 points

NAME:____________________________

Please place the letter of your selection in the blank provided. These questions are worth five points each.

5 pts 1. ___**C**___ The function $f(x) = \dfrac{x}{1-x^2}$ is

(a) continuous everywhere.
(b) discontinuous only at $x = 0$.
(c) discontinuous only at $x = 1$ and $x = -1$.
(d) discontinuous only at $x = 1$.
(e) discontinuous only at $x = 0$, $x = 1$ and $x = -1$.

Justification. The function f is an algebraic function; therefore, it will have discontinuities only at input values that produce 0 as the output from the denominator. Now, setting $1 - x^2 = 0$ gives us $x = \pm 1$ as solutions. Consequently, these are the only input values where the function f will be discontinuous.

5 pts 2. ___**E**___ The function $f(x) = \dfrac{x}{1-x^2}$ has a vertical asymptote

(a) at no value of x.
(b) only at $x = 0$.
(c) only at $x = 0$, $x = 1$, and $x = -1$
(d) only at $x = 1$.
(e) only at $x = -1$ and $x = 1$.

Justification. It is not possible to simply the formula for the function f so that either factor $1 - x$ or $1 + x$ can be cancelled from the denominator. Consequently, the function f will have a vertical asymptote at both input values.

5 pts 3. ___**D**___ The expression below is undefined when $h = 0$.

$$\frac{6h - 5h^2}{h^2 - 2h}$$

Which of the following expressions is equivalent to this expression for all nonzero values of h?

(a) $\dfrac{3-5h}{h}$
(b) $\dfrac{6h-5h}{h-2}$
(c) $\dfrac{4h-5h}{h-1}$
(d) $\dfrac{6-5h}{h-2}$
(e) More than one expression is equivalent

Justification. Two expressions are equivalent only if they produce equal output values for equal input values. For algebraic expressions, this means that the formula for one must be obtainable from the formula for the other using basic algebra techniques.

$$\frac{6h - 5h^2}{h^2 - 2h} = \frac{h(6-5h)}{h(h-2)} = \frac{6-5h}{h-2} \qquad (h \neq 0)$$

Thus, we know that Expression D is equivalent to the expression given. None of the other expressions will be equivalent. (For example, none of the other expressions listed produce the same output value as the given expression when $h = 3$.)

5 pts 4. ____**D**____ Based on your work in Problem 3, which of the following equations is true?

(a) $\lim_{h\to 0} \dfrac{6h-5h^2}{h^2-2h} = +\infty$ (b) $\lim_{h\to 0} \dfrac{6h-5h^2}{h^2-2h} = 0$

(c) $\lim_{h\to 0} \dfrac{6h-5h^2}{h^2-2h} = -4$ (d) $\lim_{h\to 0} \dfrac{6h-5h^2}{h^2-2h} = -3$

(e) $\lim_{h\to 0} \dfrac{6h-5h^2}{h^2-2h}$ does not exist

Justification. Since Expression D is equivalent to the given expression for nonzero values of h, we know that

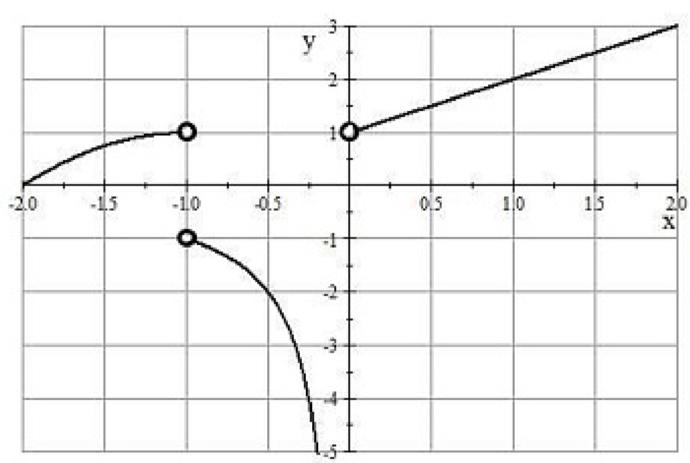
$$\lim_{h\to 0} \frac{6h-5h^2}{h^2-2h} = \lim_{h\to 0} \frac{6-5h}{h-2} = -\frac{6}{2} = -3$$

5 pts 5. ____**B**____ You place an empty glass under a faucet and turn on the tap so that water begins to fill the glass. Which of the following properly defines a *varying quantity* in this situation?

(a) the water
(b) the time passed since the faucet was turned on, measured in seconds
(c) the volume of water in the glass, measured in cubic cubic inches, when the glass is full
(d) Both (a) and (c)
(e) Both (b) and (c)

Justification. The proper definition for a quantity must include answers to these three questions (1) Precisely what is the quantity? (2) What units will be used to measure the quantity? and (3) What is the reference point for measuring the quantity. (Question 3 is not always applicable.) Of the options given, only (b) and (c) meet these criteria. Since we are concerned with the process of filling a single glass of water, the volume of water that glass will hold is constant. Therefore, only (b) describes a varying quantity.

Problems 6-8 refer to the function $y = f(x)$ whose graph is given below.



5 pts 6. $\underline{\quad\textbf{E}\quad}$According to the graph above, we have

(a) $\lim_{x \longrightarrow 0^+} f(x) = -\infty$ (b) $\lim_{x \longrightarrow 0^+} f(x) = .5$
(c) $\lim_{x \longrightarrow 0^+} f(x) = .5$ (d) $\lim_{x \longrightarrow 0^+} f(x) = -5$
(e) $\lim_{x \longrightarrow 0^+} f(x) = 1$

Justification. As the values of the input variable x get closer and closer to $x = 0$ *from the right*, the output values of the function get closer and closer to $y = 1$.

5 pts 7. $\underline{\quad\textbf{B}\quad}$According to the graph above, we have

(a) $\lim_{x \longrightarrow -1^-} f(x)$ does not exist (b) $\lim_{x \longrightarrow -1^-} f(x) = 1$
(c) $\lim_{x \longrightarrow -1^-} f(x) = -1$ (d) $\lim_{x \longrightarrow -1^-} f(x) = 0$
(e) $\lim_{x \longrightarrow -1^-} f(x) = -\infty$

Justification. As the values of the input variable x get closer and closer to $x = -1$ *from the left*, the output values of the function get closer and closer to $y = 1$.

5 pts 8. $\underline{\quad\textbf{A}\quad}$According to the graph above, the function f has a removable discontinuity

(a) at no value of x. (b) only at $x = 0$.
(c) only at $x = -1$. (d) only at $x = -1$ and $x = 0$.
(e) only at $x = 1$.

Justification. A removable discontinuity occurs at an input value where there is simply a "hole" in the graph of the function. The function f has a discontinuity at $x = -1$ and at $x = 0$. The discontinuity at $x = -1$ is a jump discontinuity, and the discontinuity at $x = 0$ is a vertical asymptote.

5 pts 9. $\underline{\quad\textbf{B}\quad}$Let x and y represent the values of two quantities, and suppose that there is a constant rate of change of $-2/3$ in the values of y with respect to the values of x. Which of the following equations is true?

(a) $y = -\frac{2}{3}x$ (b) $\Delta y = -\frac{2}{3}\Delta x$
(c) $x = -\frac{3}{2}y$ (d) Equations (a), (b), and (c) are true.
(e) Only Equations (a) and (c) are true.

Justification. To say that there is a constant rate of change of $-2/3$ in the values of y with respect to the values of x means that

$$\frac{\Delta y}{\Delta x} = -\frac{2}{3} \qquad \text{or equivalently} \qquad \Delta y = -\frac{2}{3}\Delta x$$

15 pts 10. Consider the function $y = f(x) = 2 - \sqrt{x}$.

(a) Construct the formula $y = g_9(h)$ that gives the average rate of change for the function f on the input interval from $x = 9$ to $x = 9 + h$. Do not try to simplify the formula for this function.

$$g_9(h) = \frac{f(9+h) - f(9)}{h} = \frac{\left[2 - \sqrt{9+h}\right] + 1}{h} = \frac{3 - \sqrt{9+h}}{h}$$

(b) Use the formula you constructed in Part (a) along with your graphing calculator to fill in the following table.

Value of h	-0.15	-0.10	-0.01	0.00	0.01	0.10	0.15
Value of $g_9(h)$	−0.16737	−0.16713	−0.16713	ERROR	−0.16662	−0.16620	−0.16598

(c) Based on the table values you determined in Part (b), what is the approximate local constant rate of change for the output values $f(x)$ with respect to x near $x = 9$? Briefly explain your method.

The local constant rate of change for the function f will be approximately -0.16688. I obtained this result by averaging the average rates of change for the function f on the input intervals $8.99 \le x \le 9.00$ and $9.00 \le x \le 9.01$. It would also be acceptable to take either $g_9(-0.01)$ or $g_9(0.01)$ directly as the approximation. Other approaches may also be acceptable.

10 pts 11. Without using a calculator, evaluate $\lim\limits_{x \longrightarrow +\infty} \dfrac{4x^4 - 3x + 1}{5x^4 - 8}$. You must show your steps for full credit.

$$\lim_{x \longrightarrow +\infty} \frac{4x^4 - 3x + 1}{5x^4 - 8} = \lim_{x \longrightarrow +\infty} \frac{4x^4}{5x^4} = \frac{4}{5}$$

15 pts 12. Consider the function $f(x) = \dfrac{x-3}{x^2 - 3x}$.

(a) Does this function have any discontinuities? Explain how you know.

Focusing only on the denominator, we see that $x^2 - 3x = x(x-3)$. Consequently, $x = 0$ and $x = 3$ are the roots of the denominator. These values of x will serve as the only discontinuities for the function f.

(b) Use algebra to simplify the formula for the function f as much as possible. You must show your steps for full credit.

$$f(x) = \frac{x-3}{x^2 - 3x} = \frac{x-3}{x(x-3)} = \frac{1}{x} \qquad (x \neq 3)$$

(c) Based on your work in Part (b), are there any input values where the function f has a removable discontinuity? If so, for each value $x = a$ where the function f has a removable discontinuity, determine the value of

$$\lim_{x \to a} f(x)$$

You must show your steps for full credit.

The function f will have a removable discontinuity at $x = 3$. (It will have a vertical asymptote at $x = 0$.) Now, observe

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{1}{x} = \frac{1}{3}$$

15 pts 13. Suppose that $f(x) = 2x^2 - 5$. Construct the formula for the function $g_a(h)$ thata gives the average rate of change for the function f on the input interval from $x = a$ to $x = a+h$. Use algebra to simplify your formula as much as possible. You must show your steps for full credit.

$$\begin{aligned}
g_a(h) &= \frac{f(a+h) - f(a)}{h} \\
&= \frac{\left[2(a+h)^2 - 5\right] - \left[2a^2 - 5\right]}{h} \\
&= \frac{2a^2 + 4ah + 2h^2 - 5 - 2a^2 + 5}{h} \\
&= \frac{4ah + 2h^2}{h} \\
&= 4a + 2h \qquad (h \neq 0)
\end{aligned}$$