MATH 1910 EXAM II

100 points

NAME:____________________________

Write down the derivative function for each function below. These are worth five points each.

(1) $u(x) = \tan(x)$ $\qquad$ $u'(x) = \sec^2(x)$

(2) $g(y) = \cos(y)$ $\qquad$ $g'(y) = -\sin(y)$

(3) $f(u) = u^{3/2}$ $\qquad$ $f'(u) = \frac{3}{2}\sqrt{u}$

(4) $k(a) = 4^a$ $\qquad$ $k'(a) = 4^a \ln(4)$

(5) $j(b) = b^{-3}$ $\qquad$ $j'(b) = -3b^{-4}$

Suppose f and g are differentiable functions. Match each derivative on the right with the formula used to compute it. Some of the options will not be used. These are worth five points each.

(6) ______**D**______ $\dfrac{d}{dx}[f(x)g(x)] =$

(7) ______**F**______ $\dfrac{d}{dx}[f(x) + g(x)] =$

(8) ______**B**______ $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] =$

(A) $f'(x) \cdot g'(x)$

(B) $\dfrac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

(C) $g(x)f'(x) - f(x)g'(x)$

(D) $g(x)f'(x) + f(x)g'(x)$

(E) $f'(g(x))g'(x)$

(F) $f'(x) + g'(x)$

(G) $\dfrac{f(x)g'(x) - g(x)f'(x)}{g^2(x)}$

10 pts **9.** Find the derivative of $g(x) = \tan^{3/2}(x)$. You must show your work for full credit.

Solution. Note that $g(x) = [\tan(x)]^{3/2}$. Let $u = h(x) = \tan(x)$ and let $y = f(u) = u^{3/2}$. Then

$$\begin{aligned}
\frac{dg}{dx} &= \frac{dh}{dx} \cdot \left.\frac{df}{du}\right|_{u=\tan(x)} \\
&= \frac{d}{dx}[\tan(x)] \cdot \left.\frac{d}{du}\left[u^{3/2}\right]\right|_{u=\tan(x)} \\
&= \sec^2(x) \cdot \frac{3}{2}\sqrt{\tan(x)} \\
&= \frac{3}{2}\sec^2(x)\sqrt{\tan(x)}
\end{aligned}$$

10 pts **10.** Find the derivative of $f(x) = x^2 \cos(x)$. You must show your work for full credit.

Solution. We must use the product rule to determine the formula for the derivative function. Observe

$$\begin{aligned}\frac{df}{dx} &= \frac{d}{dx}\left[x^2\right] \cdot \cos(x) + x^2 \cdot \frac{d}{dx}\left[\cos(x)\right] \\ &= 2x\cos(x) - x^2\sin(x)\end{aligned}$$

10 pts **11.** Find the derivative of $f(x) = \dfrac{2-x}{3x}$. You must show your work for full credit.

Solution. We must use the product rule to determine the formula for the derivative function. Observe

$$\begin{aligned}\frac{df}{dx} &= \left(\frac{1}{9x^2}\right)\left[3x \cdot \frac{d}{dx}\left[2-x\right] - (2-x) \cdot \frac{d}{dx}\left[3x\right]\right] \\ &= \frac{-3x - 3(2-x)}{9x^2} \\ &= -\frac{2}{3x^2}\end{aligned}$$

10 pts **12.** A function f and its derivative function are given below. What is the point-slope formula for the line tangent to the graph of the function f at the point $(2, f(2))$? You must show your steps for full credit.

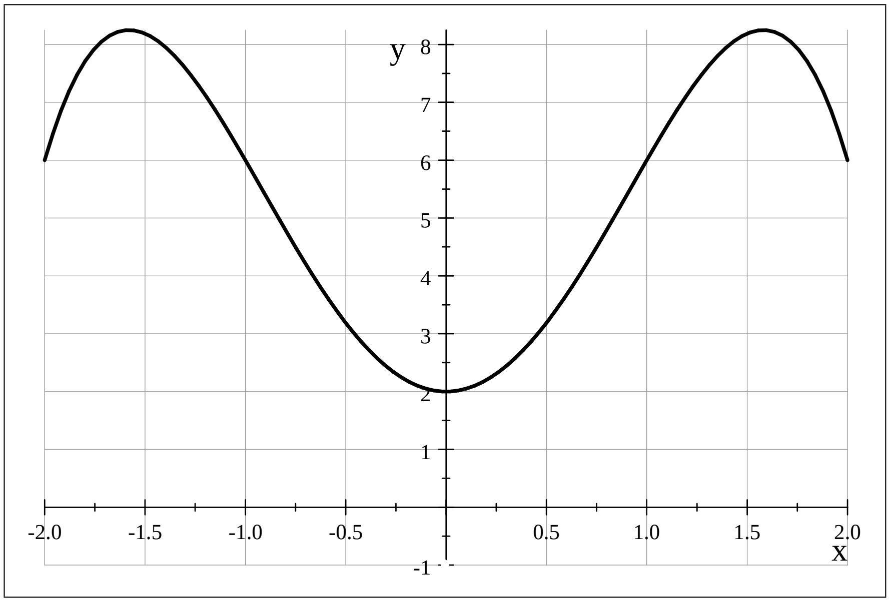
$$y = f(x) = \frac{x^2}{1+x^2} \qquad r = f'(x) = \frac{2x}{(1+x^2)^2}$$

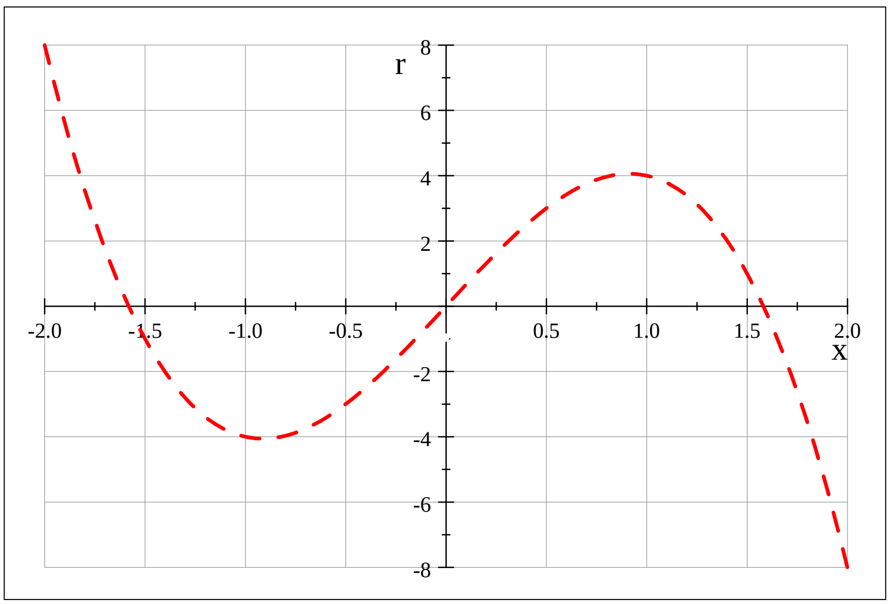
Solution. The formula for the tangent line is $y = f'(2)\left[x-2\right] + f(2)$. Now, we know

$$f(1) = \frac{(2)^2}{1+(2)^2} = \frac{4}{5} \qquad f'(1) = \frac{2(2)}{\left(1+(2)^2\right)^2} = \frac{4}{25}$$

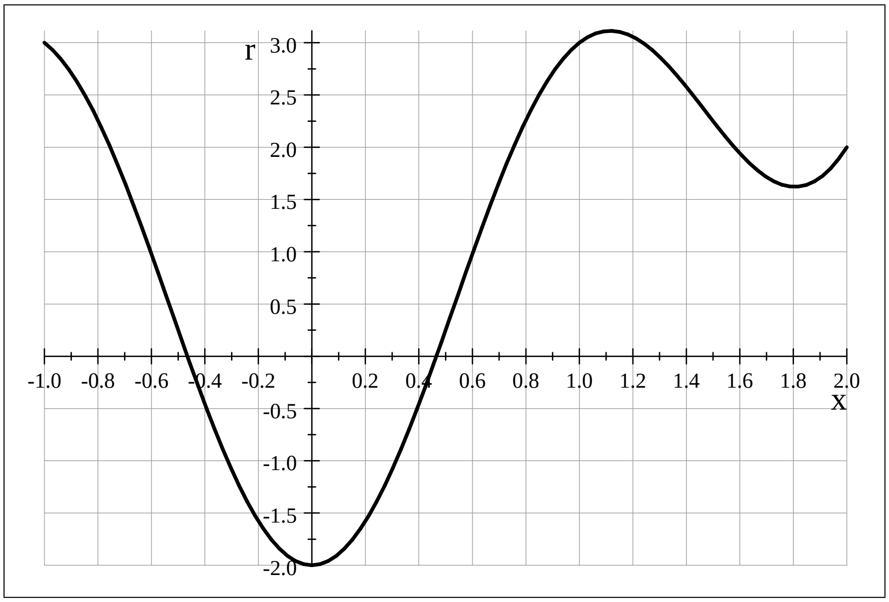
Consequently, the formula for the tangent line will be $y = \frac{4}{25}(x-2) + \frac{4}{5}$.

10 pts **13.** The graph of a function $y = f(x)$ is shown below. On the grid provided, sketch the graph of the derivative function for the function f.





The digram below shows the graph of the derivative function for a function $y = f(x)$. Problems 14 and 15 refer to this graph.



Graph of the Derivative Function for f

10 pts **14.** Based on the graph above, at what input values does the function f have local minimum output? At what input values does the function f have local maximum output?

1. (a) Local minimum outputs occur at the input values $x \approx 0.46$ (Output of f' changes from negative to positive.)

   (b) Local maximum output occur at the input values $x \approx -0.46$ (Output of f' changes from positive to negative.)

10 pts **15.** Based on the graph above, what is the approximate value of $f'(1.4)$? If we know that $f(1.4) = -2.26$, what is the point-slope formula for the tangent line to the graph of the function f at the point $(1.4, f(1.4))$?

Solution. According to the graph of the derivative function, $f'(1.4) \approx 2.5$. Hence, the formula for the tangent line is $y \approx 2.5(x - 1.4) - 2.26$.