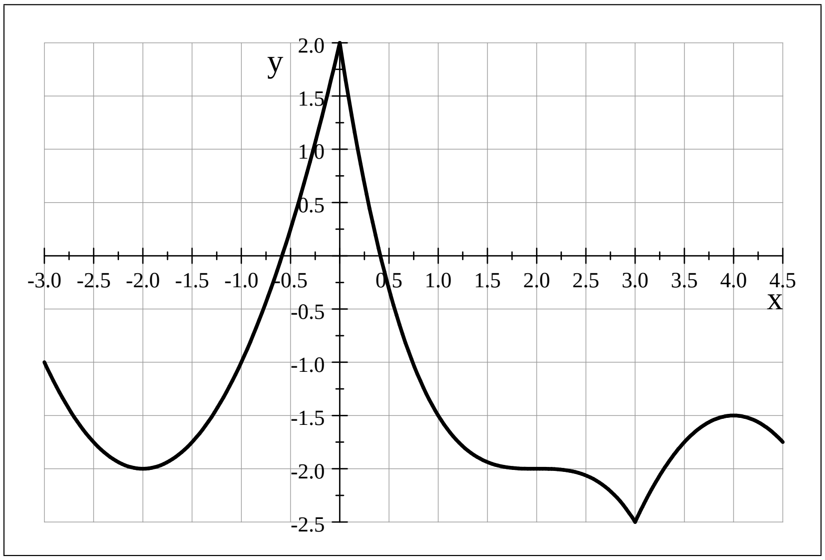
MATH 1910 EXAM III

100 points

NAME:____________________________

Problems 1 and 2 refer to the graph of the function $y = f(x)$ shown below.



10 pts 1. What are the critical numbers for the function f? Explain your answers.

Solution. The critical numbers for the function f will be those input values $x = a$ where $f(a)$ is defined and where $f'(a) = 0$ or $f'(a)$ is undefined. Based on the graph, we can see that $f'(x) = 0$ when $x = -2$, $x = 2$, and $x = 4$. We can also see that $f'(x)$ is undefined when $x = 0$ and $x = 3$.

10 pts 2. Consider the input set $1 \le x < 4.5$. At what values of x in this set does the function f have its absolute minimum and absolute maximum output values?

Solution. Based on the graph, we can see that the function f will have its absolute minimum output of -2.5 when $x = 3$. The absolute maximum output of -1.5 will occur when $x = 1$ and when $x = 4$.

10 pts 3. Construct the derivative formula for the function $y = f(x) = \dfrac{4x^3 \arcsin(x)}{3}$. You must show your steps for full credit.

$$\begin{aligned} \frac{df}{dx} &= \frac{4}{3}\frac{d}{dx}\left[x^3\right]\arcsin(x) + \frac{4x^3}{3}\frac{d}{dx}\left[\arcsin(x)\right] \\ &= \frac{4}{3}\left[3x^2\arcsin(x) + \frac{x^3}{\sqrt{1-x^2}}\right] \end{aligned}$$

4. Consider the function $y = f(x) = x^2 - 8\ln(x)$.

10 pts (a) Construct the formula for the derivative function $r = f'(x)$. Write your formula as a single fraction.

$$\frac{df}{dx} = \frac{d}{dx}\left[x^2\right] - 8\frac{d}{dx}\left[\ln(x)\right] = 2x - \frac{8}{x} = \frac{2x^2 - 8}{x}$$

10 pts (b) What are the critical numbers for the function f? You must show your work for full credit.

Solution. The derivative function is undefined when $x = 0$; however, this is *not* a critical number for the function f since $f(0)$ is also undefined. Now, $f'(x) = 0$ implies that $2x^2 - 8 = 0$. The solutions to this equation are $x = \pm 2$.

5. Consider the function $y = f(x) = \arctan(x^3 - x + 2)$.

10 pts (a) Construct the formula for the derivative function $r = f'(x)$. You must show your work for full credit.

$$\begin{aligned} \frac{df}{dx} &= \frac{d}{dx}\left[x^3 - x + 2\right] \frac{d}{du}\left[\arctan(u)\right]\Bigg|_{u=x^3-x+2} \\ &= (3x^2 - 1)\left(\frac{1}{1 + (x^3 - x + 2)^2}\right) \end{aligned}$$

10 pts (b) For what values of x will the tangent line to the function $y = f(x) = \arctan(x^3 - x + 2)$ be horizontal? You must show your work for full credit.

Solution. Setting $f'(x) = 0$ gives us the equation $3x^2 - 1 = 0$. The solutions to this equation are $x = \pm\dfrac{1}{\sqrt{3}}$.

6. Suppose that x and y are related by the formula $y^2 - x^3 = x^2 - 1$.

10 pts (a) Construct the formula for $\dfrac{dy}{dx}$. You must show your steps for full credit.

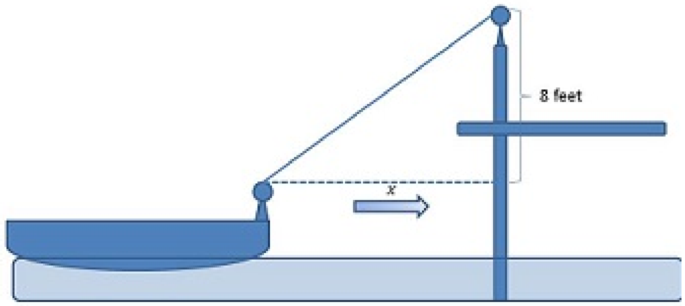
$$\begin{aligned} y^2 - x^3 = x^2 - 1 \quad &\Longrightarrow \quad \frac{d}{dx}\left[y^2 - x^3\right] = \frac{d}{dx}\left[x^2 - 1\right] \\ &\Longrightarrow \quad 2y\frac{dy}{dx} - 3x^2 = 2x \\ &\Longrightarrow \quad \frac{dy}{dx} = \frac{2x + 3x^2}{2y} \end{aligned}$$

10 pts (b) Use your answer from Part (a) to determine the point-slope formula for the tangent line to the graph of this relation at the point $(1, -1)$.

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{2(1) + 3(1)^2}{2(-1)} = -\frac{5}{2}$$

$$y = -\frac{5}{2}(x - 1) - 1$$

10 pts 7. Skipper Jack is having his motor boat towed into the dock, and the diagram below shows the setup. Let R denote the length of rope (measured in feet) between the bow of the boat and the pulley on the dock, and let x denote the horizontal distance (measured in feet) between the bow of the boat and the pulley. Assume the vertical distance (measured in feet) between the bow of the boat and the pulley remains fixed at 8 feet. Let t represent the number of seconds since the boat began to be towed. When $x = 12.2$ feet, suppose we know the length of rope is decreasing at the instantaneous rate of 2.8 feet per second. What is the corresponding instantaneous rate of change in the values of x with respect to the values of t? You must show your work for full credit.



Solution. The Pythagorean Theorem tells us that $R^2 = 64 + x^2$. Therefore, when $x = 12.2$ feet we know that $R \approx 14.6$ feet. We also know that

$$2R\frac{dR}{dt} = 2x\frac{dx}{dt} \qquad \text{or} \qquad \frac{dx}{dt} = \frac{R}{x}\frac{dR}{dt}$$

Consequently, we know that

$$\frac{dx}{dt} \approx \left(\frac{14.6 \text{ feet}}{12.2 \text{ feet}}\right)\left(-2.8\frac{\text{feet}}{\text{second}}\right) \approx -3.35\frac{\text{feet}}{\text{second}}$$