MATH 1910 PRACTICE EXAM III

5 pts 1. **D** Suppose that a function f is increasing and continuous on the half-open interval $(-2, 1]$. Which of the following statements must be true?

(a) The function f has no absolute extrema.

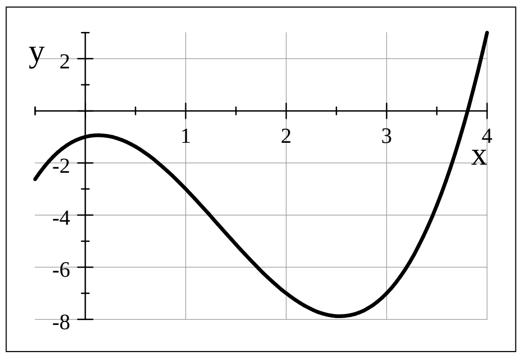
(b) The function f has an absolute minimum at $x = 1$ and an absolute maximum at $x = -2$.

(c) The function f has an absolute minimum at $x = -2$ and an absolute maximum at $x = 1$.

(d) The function f has an absolute maximum at $x = 1$ and no absolute minimum.

(e) The function f has an absolute minimum at $x = -2$ and no absolute maximum.

5 pts 2. **C** Suppose a function f is defined on the segment $(-.5, 4)$ by the graph shown below. Which of the following statements must be true?



(a) The function f has no absolute extrema.

(b) The function f has an absolute maximum and an absolute minimum.

(c) The function f has an absolute minimum but no absolute maximum.

(d) The function f has an absolute maximum but no absolute minimum.

(e) The function f has two absolute maxima but no absolute minima.

5 pts 3. **B** Suppose that f has a single critical point at $x = 3$. If $f'(0) = -2$ and $f'(4) = 6$, then we know

(a) f has no relative extrema.

(b) f has a relative minimum at $x = 3$.

(c) f has a relative maximum at $x = 3$.

(d) f has an inflection point at $x = 3$

(e) f has a horizontal tangent line at $x = 3$.

5 pts 4. **C** Suppose that $f'(2) = 0$. If $f''(2) = -5$, then we know

(a) f has no relative extremum at $x = 2$.

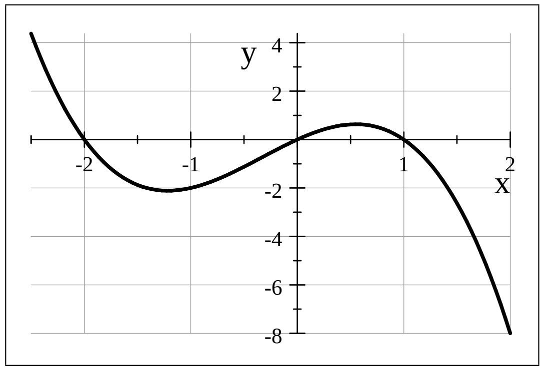
(b) f has a relative minimum at $x = 2$.

(c) f has a relative maximum at $x = 2$.

(d) f has an inflection point at $x = 2$.

(e) f has a kink in its graph at $x = 2$.

The diagram below shows the *derivative* graph for a function f. Use this graph to answer Problems 5, 6, and 7.



5 pts 5. ____**A**____ Based on the *derivative* graph shown above, we see that f has critical points at

(a) $x = -2$, $x = 0$, and $x = 1$. (b) $x = -1.25$ and $x = .6$.

(c) only $x = .6$. (d) only $x = -1.25$.

(e) $x = -2.5$ and $x = 2$.

5 pts 6. ____**A**____ Based on the *derivative* graph shown above, we see that f has relative maxima

(a) at $x = -2$ and $x = 1$. (b) only at $x = 0$.

(c) only at $x = .6$. (d) only at $x = -1.25$.

(e) at $x = -2.5$ and $x = 2$.

5 pts 7. ____**B**____ Based on the *derivative* graph shown above, we know that f will have inflection points at

(a) $x = -2$, $x = 0$, and $x = 1$. (b) $x = -1.25$ and $x = .6$.

(c) only $x = .6$. (d) only $x = -1.25$.

(e) $x = -2.5$ and $x = 2$.

Problems 8 - 10 refer to the function and its derivatives shown below.

$$f(x) = \ln\left(2x^2 - 2x + 1\right) \qquad f'(x) = \frac{2(2x-1)}{2x^2 - 2x + 1} \qquad f''(x) = \frac{8x(1-x)}{\left(2x^2 - 2x + 1\right)^2}$$

5 pts 8. ____**A**____ The function f will have critical points

(a) only at $x = 1/2$. (b) at $x = 0$ and $x = 1$.

(c) at $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$. (d) at $x = 1/2$ and $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$.

(e) at no value of x.

5 pts 9. ____**E**____ The function f will have a relative maximum

(a) only at $x = 1/2$. (b) at $x = 0$ and $x = 1$.

(c) at $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$. (d) at $x = 1/2$ and $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$.

(e) at no value of x.

5 pts 10. **C** On the interval $[1,3]$, the function f will have its absolute minimum

(a) $x=3$ (b) $x=\frac{1}{2}\left(1+\sqrt{3}\right)$

(c) $x=1$ (d) $x=1/2$

(e) $x=2$

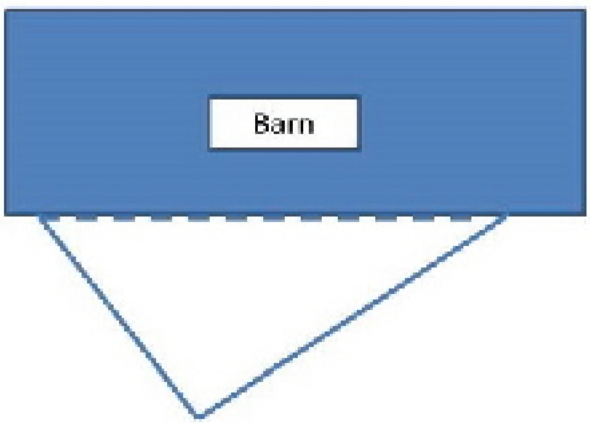
10 pts 11. One of the following limits is an indeterminate form, and one is not. Identify the indeterminate form and use L'Hôpital's Rule to compute the limit.

$$\lim_{t\longrightarrow 0}\frac{3t}{1+\cos(t)} \qquad \lim_{y\to 0}\frac{y\sin(y)}{y^2}$$

Solution. The second limit has indeterminate form $0/0$. Observe

$$\lim_{y\to 0}\frac{y\sin(y)}{y^2}\underbrace{=}_{\text{LHR}}\lim_{y\to 0}\frac{\sin(y)+y\cos(y)}{2y}\underbrace{=}_{\text{LHR}}\lim_{y\to 0}\frac{2\cos(y)-y\sin(y)}{2}=1$$

10 pts 12. The area of a right triangle is given by $A=\frac{1}{2}xy$, where x and y represent the lengths of the legs of the triangle. A rancher wants to make a corral in the form of a right triangle adjacent to her barn. The barn will serve as the hypotenuse of the triangle and therefore that side of the corral needs no fencing. If the amount of fencing she has is fixed at 100 feet, and she wants to use all of the fencing, what dimensions will maximize the area of the corral?



Solution. Let x and y represent the dimensions of the corral, measured in feet. Let A represent the area of the corral, measured in square feet. We know that the optimization formula will be

$$A=\frac{xy}{2}$$

We also know that $x>0$ and $y>0$, since these variable represent dimensions. We also know that $100=x+y$. This equation tells us that $y=100-x$; therefore, we have

$$A=f(x)=\frac{x(100-x)}{2}=\frac{100x-x^2}{2}$$

The relevant domain for this function is $0<x<100$. Now,

$$f'(x)=50-x$$

Therefore, we know that f has a single critical number, namely $x=50$. Furthermore, since $f''(x)=-1$, we know that $f''(50)<0$. Consequently, the Second Derivative Test tells us that f has its maximum output when $x=50$ feet. The dimensions of the corral will be $x=y=50$ feet.

10 pts 13. Consider the function $f(x) = x^4 - 8x^2 + 4$.

(a) Compute the first and second derivatives of f.

Solution. We know that $f'(x) = 4x^3 - 16x$, and we know that $f''(x) = 12x^2 - 16$.

(b) Identify the critical points for f. Show your work.

Solution. Setting $f'(x) = 0$ gives us $x(x^2 - 4) = 0$; therefore, f has three critical numbers, namely $x = -2$, $x = 0$, and $x = 2$.

(c) Use the Second Derivative Test to determine which of these critical points yield relative maxima or minima for f. Show your steps.

Solution. Observe that $f''(-2) = f''(2) > 0$ while $f''(0) < 0$. Consequently, f has a relative minimum output at $x = \pm 2$ and has a relative maximum output at $x = 0$.

10 pts 14. Solve the differential equation $y'' = x + \cos(x)$. You must show your steps for full credit.

Solution. Observe

$$y' = \int (x + \cos(x))\,dx \Longrightarrow y' = \frac{x^2}{2} + \sin(x) + C$$

$$y = \int \left(\frac{x^2}{2} + \sin(x) + C\right) dx \Longrightarrow y = \frac{x^3}{6} - \cos(x) + Cx + D$$

where C and D may each assume any real number as its value.

10 pts 15. Find the antiderivative family for the function $f(x) = x\left(x^2 - 5\right)^3$. You must show your work for full credit.

Solution. Let $u = x^2 - 5$ so that $\frac{du}{dx} = 2x$. Observe

$$\begin{aligned}
\int x\left(x^2 - 5\right)^3 dx &= \int \left(x^2 - 5\right)^3 [x]\,dx \\
&= \int \left(x^2 - 5\right)^3 \left[\left(\frac{1}{2}\right)(2x)\right] dx \\
&= \int u^3 \left[\left(\frac{1}{2}\right)\left(\frac{du}{dx}\right)\right] dx \\
&= \frac{1}{2}\int u^3 du \\
&= \frac{1}{8}u^4 + C \\
&= \frac{\left(x^2 - 5\right)^4}{8} + C
\end{aligned}$$