MATH 1910 PRACTICE EXAM IV

1. ____________ One antiderivative for $f(x) = e^x$ is the function

(a) $F(x) = \frac{1}{2}e^{x^2}$ (b) $F(x) = e^x - \sqrt{2}$

(c) $F(x) = \frac{1}{2}e^{2x}$ (d) $F(x) = \ln(x) + \pi$

(e) $F(x) = 3e^x$

2. ____________ In order to compute the antiderivative family for $f(x) = \dfrac{\cos(1/x)}{x^2}$ we need the substitution

(a) $u = \cos(x)$ (b) $u = \dfrac{1}{x}$

(c) $u = \dfrac{1}{x^2}$ (d) $u = x^2$

(e) $u = x$

3. ____________ The function $F(x) = x\sin(x) + 10$ is an antiderivative for which of the following functions?

(a) $f(x) = x\cos(x) + \sin(x)$ (b) $f(x) = \dfrac{x^2}{2}\sin(x) - \cos(x)$

(c) $f(x) = -\dfrac{x^2}{2}\cos(x)$ (d) $f(x) = x\cos(x)$

(e) $f(x) = \dfrac{x^2}{2}\cos(x)$

4. ____________ The second derivative for the function $y = f(x) = x\ln(x) - x$ is the function

(a) $f''(x) = 0$ (b) $f''(x) = \ln(x)$

(c) $f''(x) = \dfrac{1}{x}$ (d) $f''(x) = -\dfrac{1}{x^2}$

(e) $f''(x) = 1 + \dfrac{1}{x\ln(x)}$

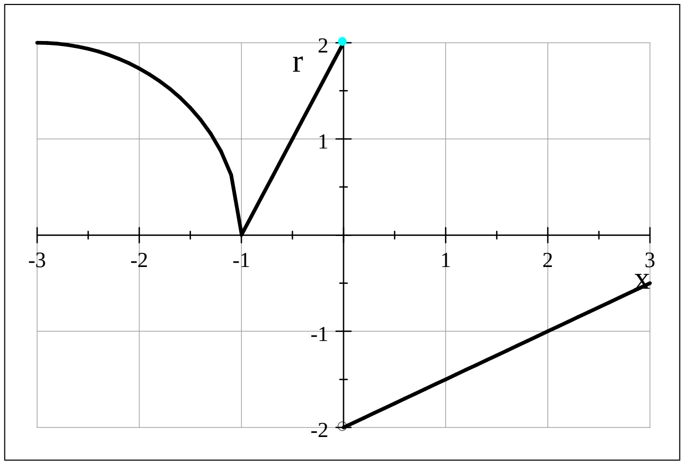
5. ____________ By making an appropriate substitution, we know that $\displaystyle\int \frac{x^2}{(1+x^3)^4}dx$

(a) is equal to $3\displaystyle\int \frac{u}{(1+u)}du$ (b) is equal to $\displaystyle\int x^2\left(\frac{1}{u^4}\right)du$

(c) is equal to $\dfrac{1}{2}\displaystyle\int \frac{u}{1+u^6}du$ (d) is equal to $\displaystyle\int u^{-4}du$

(e) is equal to $\dfrac{1}{3}\displaystyle\int u^{-4}du$

Problems 6-7 refer to the graph of a function $r = f(x)$ below. Suppose that $y = F(x)$ is an antiderivative for the function f.



6. ____________ What are the critical numbers for the function F in the viewing window shown?

(a) Critical numbers are $x = -3$ and $x = -1$ (b) Critical number is $x = -3$

(c) Critical number is $x = 0$ (d) Critical number is $x = -1$

(e) Critical numbers are $x = -1$ and $x = 0$

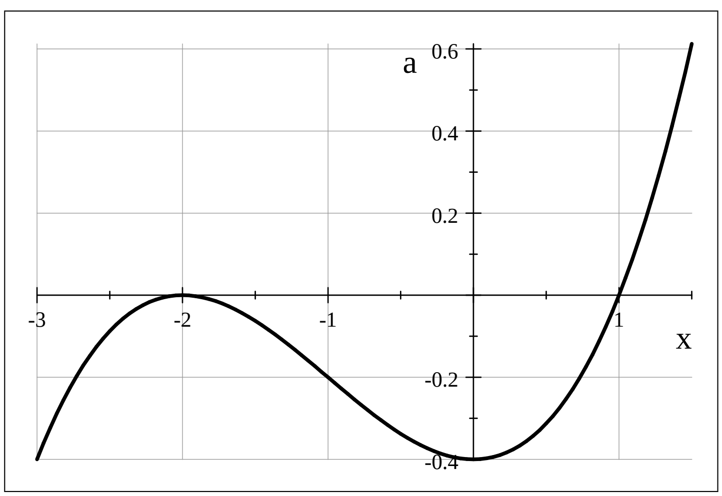
7. ____________ In the viewing window shown, at what value of x does the function F have a local minimum output?

(a) at no value of x (b) at $x = -3$

(c) at $x = 0$ (d) at $x = -1$

(e) at $x = 3$

Problems 8 and 9 refer to the graph below. This graph shows the *second derivative* function for a function $y = f(x)$.



Second Derivative Function for f

8. ____________ Based on the graph shown above, where does the function f have inflection points?

(a) $x = 1$ only (b) $x = -2$ and $x = 1$

(c) $x = 0$ only (d) $x = -1$ only

(e) no value of x

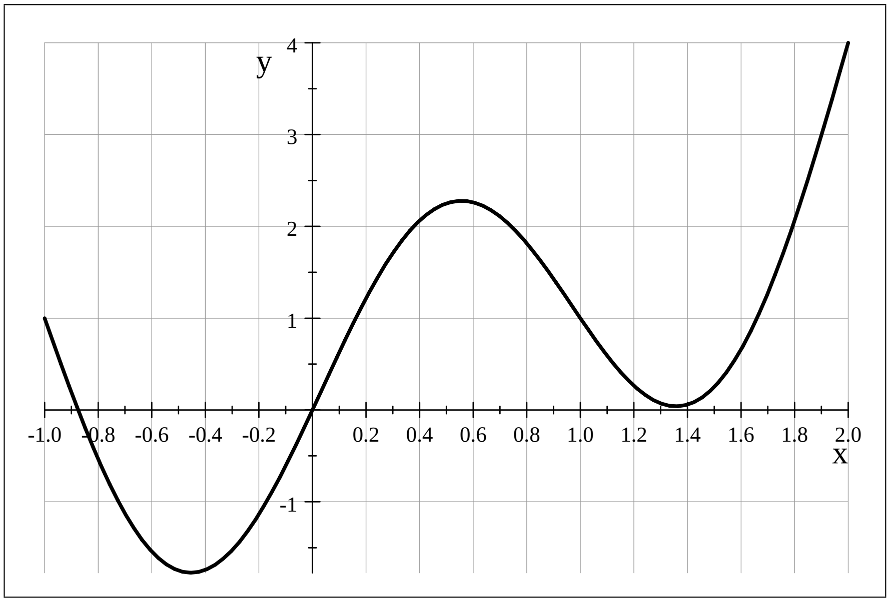
9. ____________ Based on the graph shown above, on what input intervals is the graph of the function f concave down?

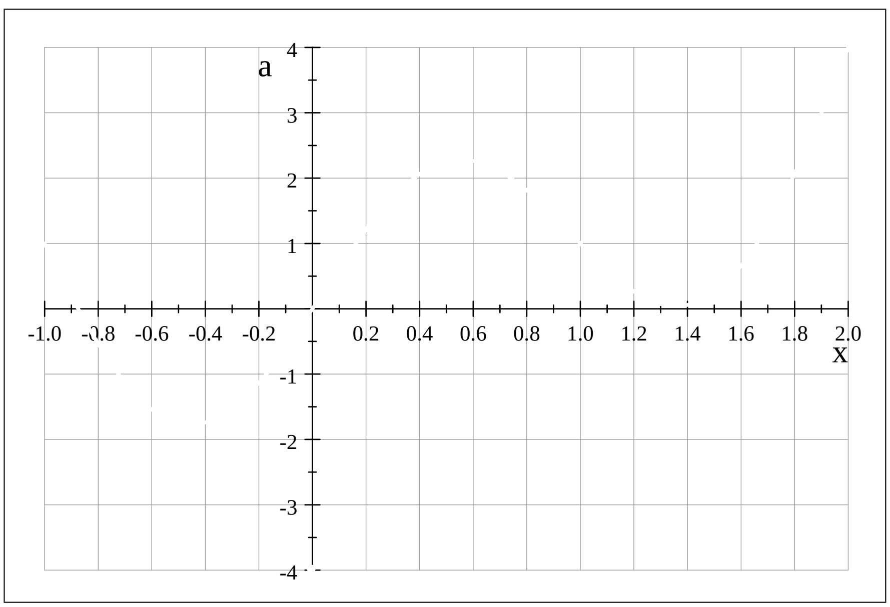
(a) $-\infty < x < -1$ (b) $-2 < x < 1$

(c) $1 < x < +\infty$ (d) $-\infty < x < 1$

(e) $-1 < x < 1$

10. The diagram below shows the graph of a function $y = f(x)$. On the grid provided, sketch the graph of the second derivative function for the function f.





11. If f is the function whose graph is shown in Problem 10, what are the critical numbers for its derivative function f'?

12. Construct the second derivative function for the function $y = f(x) = \arctan(x)$.

13. Construct the second derivative function for the function $y = f(x) = \ln(\sin(x))$.

14. Suppose that $y = f(x) = \ln(1 + x^2)$. What are the critical numbers for the function? Does the function f have any local maximum or minimum outputs?

15. Suppose that $y = f(x) = \ln(1 + x^2)$. What are the critical numbers for the derivative function $r = f'(x)$? Are any of these critical numbers inflection points for the function f?

16. Suppose that the derivative function for a function $y = f(x)$ is given below. Based on this formula, at which input values does the function f have a local maximum and/or a local minimum output?

$$f'(x) = \frac{x - 2}{x^2}$$

17. Suppose that the second derivative for a function $y = f(x)$ is given below. Based on this formula, at which input values does the *derivative* function $r = f'(x)$ have a local maximum and/or a local minimum output?

$$f''(x) = \frac{x^2 - 9}{x - 1}$$

18. Show that $F(x) = \sqrt{2} + e^x \sin(x)$ is one antiderivative for the function $f(x) = e^x(\sin(x) + \cos(x))$.

19. Show that $F(x) = x \arcsin(x) + 8$ is one antiderivative for the function $f(x) = \dfrac{\sqrt{1 - x^2}\arcsin(x) + x}{\sqrt{1 - x^2}}$.

20. Construct the antiderivative family for the function $y = f(x) = 3x + \ln(x)$.

21. Construct the antiderivative family for the function $y = f(x) = \dfrac{4\sec^2(x) - x^{1/3}}{\pi}$.

22. Evaluate $\displaystyle\int x\cos(x^2 + 1)dx$.

23. Evaluate $\displaystyle\int \frac{\sin\left(x^{-1}\right)}{x^2}dx$.

24. Construct the antiderivative family for the function $y = f(x) = \dfrac{x^2}{(x^3 + 2)^2}$.

25. Evaluate $\displaystyle\int \left(x^{-1} + 3\frac{\sec^2(x)}{\tan(x)}\right) dx$