MATH 1910 PRACTICE FINAL EXAM

Last Updated April 2012

1. ____________ The slope of the tangent line to $f(x) = \sin(x)\cos(x)$ at $x = \pi/4$ is

   (a) $m = 0$ (b) $m = 1/2$
   (c) $m = 1$ (d) $m = 1/2$
   (e) $m = -1$

2. ____________ We know that $\lim\limits_{u \longrightarrow 2} \dfrac{u^2 - 4}{(u-3)(u-2)^2}$

   (a) is equal to 0 (b) is equal to -4
   (c) is equal to $+\infty$ (d) is equal to $-\infty$
   (e) does not exist

3. ____________ If $f(x) = \sqrt{1+x^2}$, then which of the following limits correctly represents $f'(2)$?

   (a) $f'(2) = \lim\limits_{\triangle x \longrightarrow 0} \dfrac{\sqrt{5 + (\triangle x)^2} - \sqrt{5}}{\triangle x}$ (b) $f'(2) = \lim\limits_{\triangle x \longrightarrow 0} \dfrac{2 - \sqrt{1 + (2 + \triangle x)^2}}{\triangle x}$

   (c) $f'(2) = \lim\limits_{\triangle x \longrightarrow 0} \dfrac{\sqrt{3 + (\triangle x)^2} - 2}{\triangle x\,(3 + (\triangle x)^2)}$ (d) $f'(2) = \lim\limits_{\triangle x \longrightarrow 0} \dfrac{\sqrt{1 + (2 + \triangle x)^2} - \sqrt{5}}{\triangle x}$

   (e) $f'(2) = \lim\limits_{\triangle x \longrightarrow 0} \dfrac{(2 + \triangle x)^2 - 4}{\triangle x \left(\sqrt{1 + (2 + \triangle x)^2} - \sqrt{5}\right)}$

4. ____________ The equation of the tangent line to $f(x) = x^{2/3} + x + 1$ at the point $(1, 3)$ is given by

   (a) $y = \dfrac{5}{3}[x-1] + 3$ (b) $y = \left(\dfrac{2 + \sqrt[3]{x}}{3\sqrt[3]{x}}\right)[x-1] + 3$

   (c) $y = \left(\dfrac{2 + \sqrt[3]{x}}{3\sqrt[3]{x}}\right)[x-3] + 1$ (d) $y = \dfrac{2}{3}[x-3] + 1$

   (e) $y = 3\left[x - \dfrac{5}{3}\right] + 1$

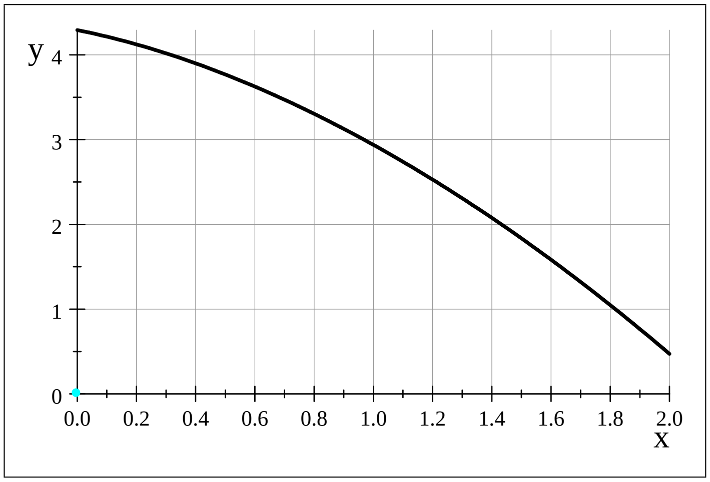
5. ____________ Suppose a farmer wants to construct a rectangular corral of maximum area using 1000 feet of fencing. If L and W denote the length and width of the corral, then the optimization formula as a function of W will be

   (a) $f(W) = \dfrac{1000}{W}(500 - W)$ (b) $f(W) = 500W - W^2$

   (c) $f(W) = \dfrac{2000}{W} + 2W$ (d) $f(W) = W^2 + 1000$

   (e) $f(W) = 1000W - \dfrac{2}{W}$

Problems 6 and 7 refer to the graph of the function f shown below.



6. ____________ Based on the graph above, we know that on the open interval $(0, 2)$

(a) $f'(x) < 0$ and $f''(x) < 0$ (b) $f'(x) < 0$ and $f''(x) > 0$

(c) $f'(x) > 0$ and $f''(x) < 0$ (d) $f'(x) > 0$ and $f''(x) > 0$

(e) $f'(x) < 0$ and $f''(x) = 0$

7. ____________ The slope of the tangent line to f at $x = 1$ is

(a) $m \approx -1$ (b) $m \approx -.5$

(c) $m \approx -2$ (d) $m \approx -.4$

(e) $m \approx -.22$

8. ____________ For the curve $xy = \cos(y)$, we know that

(a) $\frac{dy}{dx} = \frac{x - \sin(y)}{y}$ (b) $\frac{dy}{dx} = -\frac{y}{x + \sin(y)}$

(c) $\frac{dy}{dx} = \frac{x + \sin(y)}{x}$ (d) $\frac{dy}{dx} = \frac{x + \sin(y)}{y}$

(e) $\frac{dy}{dx} = \frac{\sin(y) - x}{y}$

9. ____________ Suppose that f is a twice-differentiable function on the open interval (a, b). If $f'(x) > 0$ and $f''(x) < 0$ on this interval, then we know f is

(a) decreasing and concave up (b) decreasing and concave down

(c) increasing and concave up (d) increasing and concave down

(e) none of the above

10. ____________ If $f(t) = \cos(t^3)$, then we know

(a) $f'(t) = -\sin(t^3)$ (b) $f'(t) = -3t^2 \sin(t^3)$

(c) $f'(t) = 3t^2 \cos(t) - t^3 \sin(t)$ (d) $f'(t) = t^2(\sin(t) - \cos(t^3))$

(e) $f'(t) = 3t^2 \cos(t)$

11. ____________ Suppose we know that $g(x) = 2f'(x)$. In this case, we also know $\int_0^3 f(x)g(x)dx$

(a) is equal to $\frac{3}{4}$ (b) is equal to $\frac{9}{2}$

(c) is equal to $\frac{(f(3)-f(0))^2}{2}$ (d) is equal to $\frac{9}{4}$

(e) is equal to $f(3)^2 - f(0)^2$

12. ____________ After applying the appropriate substitution in $\int_{x=1}^{x=4} \frac{\sin(2+\ln(x))}{x}dx$, the new limits will be

(a) $u = 1$ and 4 (b) $u = 1$ and $u = 1/4$
(c) $u = 2$ and $u = 2 + \ln(4)$ (d) $u = \sin 2$ and $u = \sin(2 + \ln(4))$
(e) $u = 1$ and $u = 2$

13. ____________ If $F(t) = \int_1^t x\sqrt{x^2-1}dx$, then $F'(3)$

(a) is equal to $\sqrt{3}$ (b) is equal to $2\sqrt{2}$

(c) is equal to $\frac{2\sqrt{2}}{3}$ (d) is equal to $\frac{2\sqrt{3}}{3}$

(e) is equal to $6\sqrt{2}$

14. ____________ The second derivative of $f(x) = 3^x + \ln(x)$ will be

(a) $f'(t) = x(x-1)3^{x-2} + \frac{1}{x}$ (b) $f'(t) = x(x-1)3^{x-2} - \ln(x)$

(c) $f'(t) = 3^x \ln^2(3) - \frac{1}{x^2}$ (d) $f'(t) = 3^x \ln(9) + \frac{1}{x}$

(e) $f'(t) = 3^x - \frac{1}{x^2}$

15. ____________ If a closed rectangular box with dimensions x, y, and z of minimum surface area has a square bottom and fixed volume of 1000 cubic inches, then its optimization formula as a function of x is

$$f(x) = \frac{2x^3 + 4000}{x}$$

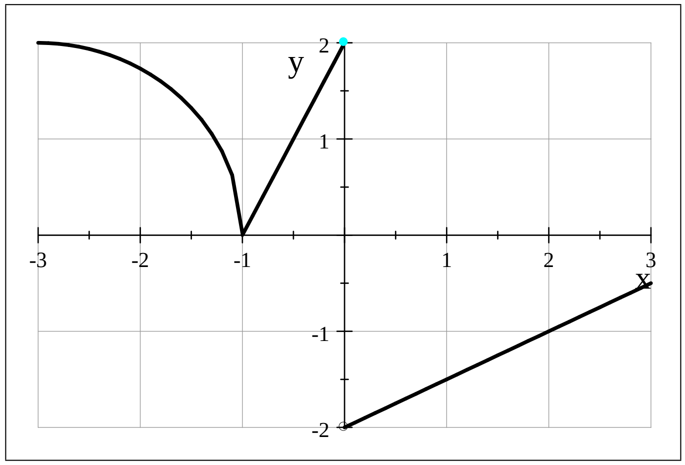
The relevant domain for the optimization function will be

(a) $0 \leq x < +\infty$ (b) $0 < x < +\infty$

(c) $0 < x \leq 1000$ (d) $0 \leq x \leq 10\sqrt[3]{2}$

(e) $0 < x < 10\sqrt[3]{2}$

Problems 16-18 refer to the graph of a function f below. The curve is an arc of a circle of radius 2.



16. ____________ Based on this diagram, we know $\int_{-3}^{2} f(x)dx$

(a) is equal to $\frac{4\pi - 7}{4}$ (b) is equal to $\pi - 1$

(c) is equal to $\frac{\pi}{4} - 3$ (d) is equal to $\pi - 2$

(e) is equal to $3 - \frac{\pi}{4}$

17. ____________ If $F(x) = \int_{0}^{x} f(t)dt$, then $F(-1)$

(a) is equal to 1 (b) is equal to 0

(c) is equal to $\frac{1}{2}$ (d) is equal to $-\frac{1}{2}$

(e) is equal to -1

18. ____________ If $G(x) = \int_{1}^{x} f(t)dt$, then G will have a relative minimum at

(a) $x = 0$ only (b) $x = -1$ only

(c) $x = -3$ only (d) $x = -3$ and $x = 0$

(e) no value of x

19. ____________ If $f(x) = \ln(1 + x^3)$, then the equation of the tangent line to f at $x = 1$ will be

(a) $y = \ln(2)[x - 1] + \frac{1}{3}$ (b) $y = \frac{3x^2}{1 + x^3}[x - 1] + \ln(2)$

(c) $y = \frac{1}{3}[x - \ln(2)] + 1$ (d) $y = \frac{3}{2}[x - 1] + \ln(2)$

(e) $y = \ln(2)\left[x - \frac{2}{3}\right] + 1$

20. ____________ If $x = \arcsin(y^2)$, then differentiating with respect to t gives us

(a) $\dfrac{dx}{dt} = \dfrac{y^4}{1+y^4}$ (b) $\dfrac{dx}{dt} = \dfrac{1}{1+y^4}\dfrac{dy}{dt}$

(c) $1 - \dfrac{1}{\sqrt{1-y^4}}\dfrac{dy}{dt} = 0$ (d) $\dfrac{dy}{dt} = \dfrac{\sqrt{1-y^4}}{2y}\dfrac{dx}{dt}$

(e) $\dfrac{dy}{dt} = \dfrac{y}{1-y^2}\dfrac{dx}{dt}$

21. ____________ Which of the following is an antiderviative for $f(t) = \ln(t)$?

(a) $F(t) = 8 + \dfrac{1}{t}$ (b) $F(t) = t\ln(t) - t + 10$

(c) $F(t) = \ln(t) + t$ (d) $F(t) = e^t - 6$

(e) $F(t) = \dfrac{\ln^2(t)}{2}$

22. ____________ The function $F(x) = 2x\cos(x^2) - 8$ is an antiderivative for the function

(a) $f(x) = 2\cos(x^2) - 4x^2\sin(x^2)$ (b) $f(x) = -2\sin(x^2)$

(c) $f(x) = \sin(x^2) - 8x + 5$ (d) $f(x) = \sin(x^2)$

(e) $f(x) = 4x^2\sin(x^2) - 8x$

23. ____________ Which of the following are critical points for $f(x) = x + x^{2/3} - 8$?

(a) There are no critical points. (b) Both $x = 1$ and $x = -1$

(c) Only $x = 0$ (d) Both $x = 0$ and $x = -8/27$

(e) Both $x = 1$ and $x = 1/2$

24. ____________ What is the exact value of $\displaystyle\int_{-1}^{1}(x^2 + 2x)dx$?

(a) 2/3 (b) 2

(c) 4 (d) 0

(e) 3/2

25. ____________ What is the value of $\displaystyle\lim_{x \longrightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$?

(a) 1 (b) 0

(c) 1/2 (d) 3/4

(e) Does not exist

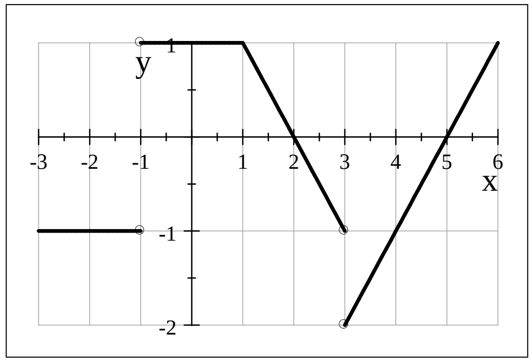
26. ____________ The second derivative of $f(x) = e^{-x^2}$ is

(a) $f''(x) = -2xe^{-x^2}$ (b) $f''(x) = 2e^{-x^2}(2x^2 - 1)$

(c) $f''(x) = 4e^{-x^2}$ (d) $f''(x) == xe^{-x^2}(2 + x)$

(e) $f''(x) = e^{-x^2}$

Problems 27-31 refer to the graph of the function f shown below.



27. ____________ According to the graph, $\lim_{x \longrightarrow 3^-} f(x)$

(a) is equal to -3 (b) is equal to -1
(c) is equal to -2 (d) is equal to -2.5
(e) Does not exist

28. ____________ According to the graph, $\lim_{x \longrightarrow -1} f(x)$

(a) is equal to 1 (b) is equal to -1
(c) is equal to 0 (d) is equal to -2
(e) Does not exist

29. ____________ The exact value of $\int_{-1}^{3} f(x)dx$ is

(a) 1/2 (b) 4
(c) 2 (d) 0
(e) 3

30. ____________ Suppose that F is an antiderivative for f. At which points will F have a relative minimum?

(a) At $x = 1$ (b) At $x = 2$
(c) At $x = 1$ and $x = 3$ (d) At $x = 2$ and $x = 5$
(e) At $x = -1$ and $x = 5$

31. ____________ Suppose that $F = \int_{4}^{t} f(x)dx$. What is the value of F at the point where its relative maximum occurs?

(a) 2 (b) 0
(c) 1/2 (d) $-3/2$
(e) 3/2

32. ____________ The net area between the graph of $f(x) = 4x^3 - 1$ and the x-axis on the interval $[0, 2]$ is

(a) 12 (b) 3.5

(c) 14 (d) 7

(e) 0

33. ____________ The slope of the tangent line to the curve $\sqrt{x} - x\sqrt{y} = -1$ at the point $(1, 4)$ is

(a) $m = -12$ (b) $m = -6$

(c) $m = 1/12$ (d) $m = 1/6$

(e) undefined

34. ____________ What is the exact value of $\int_{-1}^{1} \frac{3x}{x^2+1} dx$?

(a) 0 (b) 1

(c) $\frac{3}{2}\ln(2)$ (d) $\frac{\pi}{2}$

(e) $\frac{3}{2}$

35. ____________ Which of the following statements is true of $f(x) = \arctan(x)$?

I The function f has critical points at $x = \pm 1$.

II The function f has a relative maximum at $x = 1$.

III The function f has a relative minimum at $x = -1$.

(a) Only Statement I. (b) Only Statements I and II.

(c) Only Statements I and III. (d) All are true.

(e) None are true.

36. ____________ Suppose that f is a twice-differentiable function. If $f'(3) = 0$ and $f''(3) = 10$, then which of the following statements is true?

I The function f has an inflection point at $x = 3$.

II The function f is concave up at $x = 3$.

III The function f has a relative minimum at $x = 3$.

(a) Only Statement I. (b) Only Statement II.

(c) Only Statements I and III. (d) Only Statements II and III.

(e) None are true.

37. ____________ At which values of x is the tangent line to $f(x) = x^3 + 3x - 5$ parallel to the line $y = 15x + 4$?

(a) $x = \pm 2$ (b) $x = \pm\sqrt{3}$

(c) $x = 0$ (d) $x = 5$

(e) $x = \sqrt{5}$

38. ____________ The absolute extrema for the function $f(x) = 4 - 5x + x^5$ on the interval $[-3, 2]$ occur at

(a) $x = -3$ and $x = -1$ (b) $x = -3$ and $x = 2$

(c) $x = -1$ and $x = 1$ (d) $x = 1$ and $x = 2$

(e) $x = -3$ and $x = 1$

39. ____________ The appropriate substitution in the indefinite integral $\int \frac{\sqrt{x}}{\cos^2(x\sqrt{x})}dx$ would be

(a) $x = \sqrt{x}$ (b) $u = x$
(c) $u = \cos^2(x\sqrt{x})$ (d) $u = \cos(x\sqrt{x})$
(e) $u = x\sqrt{x}$

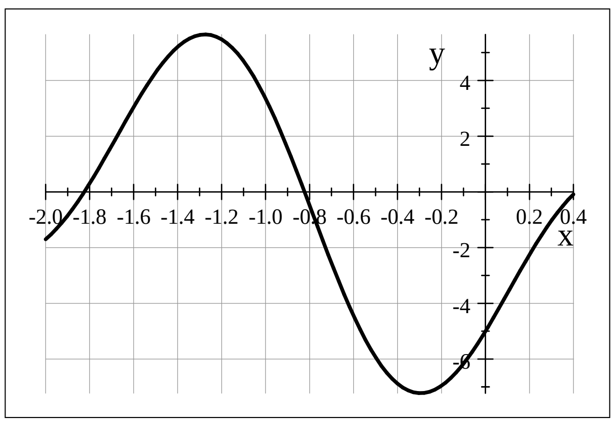
40. ____________ When the appropriate substitution has been made for $\int_0^2 \frac{x^3+1}{(x^4+4x+1)^3}dx$ the new limits will be

(a) $u = 0$ and $u = 2$ (b) $u = 1$ and $u = 25$

(c) $u = 1$ and $u = 9$ (d) $u = 0$ and $x = 16$

(e) $u = 0$ and $u = 8$

Problems 41-45 refer to the graph of the function f shown below.



41. ____________ Based on the graph above, the points in the open interval $(-2, .4)$ guaranteed by the Mean Value Theorem occur at

(a) $x \approx -1.83$ and $x \approx -.83$ (b) $x \approx -1.83$ and $x \approx .37$

(c) $x \approx -1.35$ and $x \approx -.25$ (d) $x \approx -1.60$ and $x \approx .10$

(e) $x \approx -1.22$ and $x \approx -.40$

42. ____________ On the half-open interval $(-1.8, 0]$, we know that f has

(a) no critical points (b) an absolute minimum only

(c) an absolute maximum only (d) an absolute maximum and minimum

(e) no absolute maximum or minimum

43. ____________ At which of the following points is $f'(x)$ clearly positive?

(a) $x = -1$ (b) $x = .1$

(c) $x = -.6$ (d) $x = -.3$

(e) $x = -.8$

44. ____________ At which of the following points is $f''(x)$ clearly positive?

(a) $x = -1.3$ (b) $x = -1$

(c) $x = .3$ (d) $x = -.3$

(e) $x = -.8$

45. ____________ If F is an antiderivative for f, then we know that F has inflection points at

(a) $x \approx -1.83$, $x \approx -.83$, and $x \approx .10$ (b) $x \approx -1.83$, $x \approx -.83$, and $x \approx .10$

(c) $x \approx -1.35$ and $x \approx -.25$ (d) $x \approx -1.60$ and $x \approx .10$

(e) $x \approx -1.30$ and $x \approx -.30$

Problems 46-48 refer to the function and its derivatives given below.

$$f(x) = \ln\left(4x^2 - x + 1\right) \qquad f'(x) = \frac{8x - 1}{4x^2 - x + 1} \qquad f''(x) = -\frac{48x^2 - 8x + 1}{\left(4x^2 - x + 1\right)^2}$$

46. ____________ The function f has critical points at

(a) $x = 1/8$ (b) $x = \dfrac{1 \pm \sqrt{15}}{8}$

(c) $x = 1/8$ and $x = \dfrac{1 \pm \sqrt{15}}{8}$ (d) $x = 0$

(e) There are no critical points.

47. ____________ The function f has inflection points at

(a) $x = \dfrac{1 \pm 4\sqrt{2}}{12}$ (b) $x = \dfrac{1 \pm \sqrt{15}}{8}$

(c) $x = \dfrac{1 \pm 4\sqrt{2}}{12}$ and $x = \dfrac{1 \pm \sqrt{15}}{8}$ (d) $x = 1/8$

(e) There are no inflection points.

48. ____________ The value of f at its relative minimum will be approximately

(a) $-.06454$ (b) $.62861$

(c) $.51653$ (d) -1.3863

(e) The function f does not have a relative minimum.

49. ____________ A closed circular cylinder of radius R and height H must have a surface area of 2000 square inches. If we wish to maximize the volume of this cylinder, what will its optimization formula be as a function of R? (The surface area of a cylinder is $S = 4\pi RH$, and the volume is $V = \pi R^2 H$.)

(a) $f(R) = 500R$

(b) $f(R) = 2\left(\dfrac{\pi R^3 + 4000}{R}\right)$

(c) $f(R) = \dfrac{1000R - \pi R^2}{2}$

(d) $f(R) = \dfrac{8000}{R}$

(e) $f(R) = 2\left(\dfrac{4000 - \pi R^3}{R}\right)$

50. ____________ A wall rises vertically from a patch of level ground, and a ten-foot ladder is leaning against this wall. If the top of the ladder is sliding down the wall at a constant speed of k feet per second, how fast is the base of the ladder moving when the top is b feet from the ground?

(a) $\dfrac{dx}{dt} = \dfrac{1}{\sqrt{100-b^2}}(50 - bk)$ feet per second

(b) $\dfrac{dx}{dt} = \dfrac{50-b}{\sqrt{100-b^2}}k$ feet per second

(c) $\dfrac{dx}{dt} = -\dfrac{bk}{\sqrt{100-b^2}}$ feet per second

(d) $\dfrac{dx}{dt} = b\left(50 - \sqrt{100-b^2}k\right)$ feet per second

(e) $\dfrac{dx}{dt} = \dfrac{\sqrt{100-b^2}}{b}k$ feet per second

ANSWERS

1) A	11) C	21) B	31) A	41) C
2) E	12) C	22) A	32) C	42) D
3) D	13) E	23) D	33) B	43) B
4) A	14) C	24) A	34) A	44) D
5) B	15) B	25) D	35) E	45) E
6) A	16) D	26) B	36) D	46) A
7) C	17) E	27) B	37) A	47) E
8) B	18) E	28) E	38) B	48) A
9) D	19) D	29) C	39) E	49) C
10) B	20) D	30) E	40) B	50) C