MATH 1910 PRACTICE FINAL EXAM

Last Updated April 2017

1. _____ The slope of the tangent line to $f(x) = \sin(x)\cos(x)$ at $x = \pi/4$ is

   (a) $m = 0$ (b) $m = 1/2$

   (c) $m = 1$ (d) $m = 1/2$

   (e) $m = -1$

2. _____ We know that $\lim_{u \longrightarrow 2^+} \dfrac{u^2 - 4}{(u-3)(u-2)^2}$

   (a) is equal to 0 (b) is equal to -4

   (c) is equal to $+\infty$ (d) is equal to $-\infty$

   (e) does not exist

3. _____ If $f(x) = \sqrt{1+x^2}$, then which of the following limits correctly represents $f'(2)$?

   (a) $f'(2) = \lim_{h \longrightarrow 0} \dfrac{\sqrt{5+h^2} - \sqrt{5}}{h}$ (b) $f'(2) = \lim_{h \longrightarrow 0} \dfrac{2 - \sqrt{1+(2+h)^2}}{h}$

   (c) $f'(2) = \lim_{h \longrightarrow 0} \dfrac{\sqrt{3+h^2} - 2}{h\,(3+h^2)}$ (d) $f'(2) = \lim_{h \longrightarrow 0} \dfrac{\sqrt{1+(2+h)^2} - \sqrt{5}}{h}$

   (e) $f'(2) = \lim_{h \longrightarrow 0} \dfrac{(2+h)^2 - 4}{h\left(\sqrt{1+(2+h)^2} - \sqrt{5}\right)}$

4. _____ The equation of the tangent line to $f(x) = x^{2/3} + x + 1$ at the point $(1, 3)$ is given by

   (a) $y = \dfrac{5}{3}[x-1] + 3$ (b) $y = \left(\dfrac{2 + \sqrt[3]{x}}{3\sqrt[3]{x}}\right)[x-1] + 3$

   (c) $y = \left(\dfrac{2 + \sqrt[3]{x}}{3\sqrt[3]{x}}\right)[x-3] + 1$ (d) $y = \dfrac{2}{3}[x-3] + 1$

   (e) $y = 3\left[x - \dfrac{5}{3}\right] + 1$

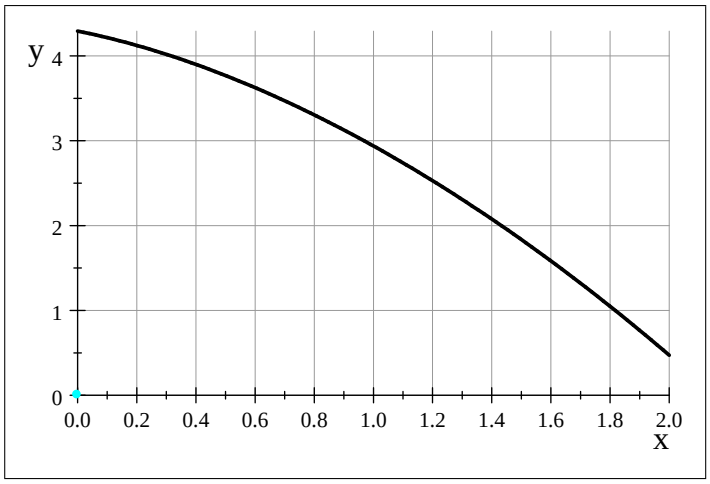
5. _____ If $f(x) = xe^x$, then what is the formula for the second derivative of f?

   (a) $f''(x) = (1 + x\ln(x))e^x$ (b) $f''(x) = 1 + \ln(x)$

   (c) $f''(x) = xe^x + 1$ (d) $f''(x) = e^x$

   (e) $f''(x) = (2+x)e^x$

Problems 6 and 7 refer to the graph of the function f shown below.



6. _____ Based on the graph above, we know that on the input interval $0 < x < 2$ we have

(a) $f'(x) < 0$ and $f''(x) < 0$ (b) $f'(x) < 0$ and $f''(x) > 0$

(c) $f'(x) > 0$ and $f''(x) < 0$ (d) $f'(x) > 0$ and $f''(x) > 0$

(e) $f'(x) < 0$ and $f''(x) = 0$

7. _____ The slope of the tangent line to the graph of the function f at $x = 1$ is

(a) $m \approx -1$ (b) $m \approx -.5$

(c) $m \approx -2$ (d) $m \approx -.4$

(e) $m \approx -.22$

8. _____ For the curve $x^2 = y\cos(x)$, we know that

(a) $\dfrac{dy}{dx} = 2x + y\sin(x)$ (b) $\dfrac{dy}{dx} = \dfrac{2x + y\sin(x)}{\cos(x)}$

(c) $\dfrac{dy}{dx} = \sec(x) + y\tan(x)$ (d) $\dfrac{dy}{dx} = \dfrac{x + \sin(y)}{y}$

(e) $\dfrac{dy}{dx} = \dfrac{\sin(y) - x^2}{y}$

9. _____ Suppose that f is a twice-differentiable function on the input interval $a < x < b$. If $f'(x) > 0$ and $f''(x) < 0$ on this interval, then we know the graph of f is

(a) decreasing and concave up (b) decreasing and concave down

(c) increasing and concave up (d) increasing and concave down

(e) none of the above

10. _____ If $f(t) = \cos(t^3)$, then we know

(a) $f'(t) = -\sin(t^3)$ (b) $f'(t) = -3t^2\sin(t^3)$

(c) $f'(t) = 3t^2\cos(t) - t^3\sin(t)$ (d) $f'(t) = t^2(\sin(t) - \cos(t^3))$

(e) $f'(t) = 3t^2\cos(t)$

11. _____ Suppose we know that $g(x) = 2f'(x)$. In this case, we also know $\int_0^3 f(x)g(x)dx$

(a) is equal to $\frac{3}{4}$ (b) is equal to $\frac{9}{2}$

(c) is equal to $f(3)^2 - f(0)^2$ (d) is equal to $\frac{9}{4}$

(e) is equal to $\frac{f(3)^2 - f(0)^2}{4}$

12. _____ The appropriate substitution to use in evaluating $\int \frac{\sin(2+\ln(x))}{x}dx$ would be

(a) $u = \sin(2 + \ln(x))$ (b) $u = 1/x$

(c) $u = 2 + \ln(x)$ (d) $u = \sin(x)$

(e) $u = \ln(x)$

13. _____ If we let $F(t) = \int_5^t x\sqrt{x^2-1}dx$, then

(a) $F(t) = 10 - \sqrt{t^2-1}$ (b) $F(t) = \frac{t^2+t-1}{\sqrt{t^2-1}}$

(c) $F(t) = 5 - \frac{1}{3}\left(t^2-1\right)^{3/2}$ (d) $F(t) = \sqrt{t^2-1} - 10$

(e) $F(t) = \frac{\left(t^2-1\right)^{3/2}}{3} - 16\sqrt{6}$

14. _____ The second derivative of $f(x) = 3^x + \ln(x)$ will be

(a) $f'(t) = x(x-1)3^{x-2} + \frac{1}{x}$ (b) $f'(t) = x(x-1)3^{x-2} - \ln(x)$

(c) $f'(t) = 3^x \ln^2(3) - \frac{1}{x^2}$ (d) $f'(t) = 3^x \ln(9) + \frac{1}{x}$

(e) $f'(t) = 3^x - \frac{1}{x^2}$

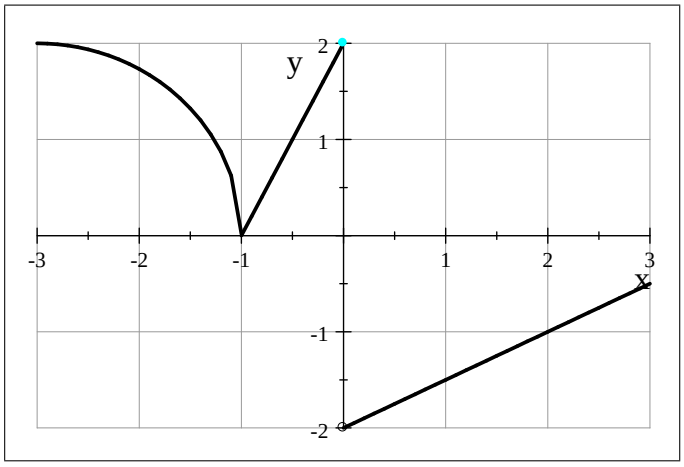
15. _____ What is the derivative function for $f(x) = \frac{\sin(x)}{1-\cos(x)}$?

(a) $\frac{df}{dx} = -\frac{\cos(x)}{(1-\cos(x))^2}$ (b) $\frac{df}{dx} = -\frac{1}{1-\cos(x)}$

(c) $\frac{df}{dx} = \frac{\sin(x)-1}{(1-\cos(x))^2}$ (d) $\frac{df}{dx} = \frac{\cos(x)}{(1-\cos(x))^2}$

(e) $\frac{df}{dx} = \frac{\sin(x)\cos(x)}{(1-\cos(x))^2}$

Problems 16-18 refer to the graph of a function f below. The curve is an arc of a circle of radius 2.



16. _____ Based on this diagram, we know $\int_{-3}^{2} f(x)dx$

(a) is equal to $\dfrac{4\pi - 7}{4}$ (b) is equal to $\pi - 1$

(c) is equal to $\dfrac{\pi}{4} - 3$ (d) is equal to $\pi - 2$

(e) is equal to $3 - \dfrac{\pi}{4}$

17. _____ If $F(x) = \int_{-1}^{x} f(t)dt$, then $F(0)$

(a) is equal to -1 (b) is equal to 0

(c) is equal to 2 (d) is equal to $-\dfrac{1}{2}$

(e) is equal to 1

18. _____ If the function f is the derivative of a function G, then the function G will have a local maximum output at

(a) $x = 0$ only (b) $x = -1$ only

(c) $x = -3$ only (d) $x = -3$ and $x = 0$

(e) no value of x

19. _____ If $f(x) = \ln(1 + x^3)$, then the equation of the tangent line to the graph of f at $x = 1$ will be

(a) $y = \ln(2)[x - 1] + \dfrac{1}{3}$ (b) $y = \dfrac{3x^2}{1 + x^3}[x - 1] + \ln(2)$

(c) $y = \dfrac{1}{3}[x - \ln(2)] + 1$ (d) $y = \dfrac{3}{2}[x - 1] + \ln(2)$

(e) $y = \ln(2)\left[x - \dfrac{2}{3}\right] + 1$

20. _____ If $x = \sin(y^2)$, then differentiating with respect to t gives us

(a) $1 = 2y\cos\left(y^2\right)$ (b) $\dfrac{dx}{dt} = \sin\left(y^2\right)$

(c) $\dfrac{dy}{dt} = \dfrac{\csc\left(y^2\right)}{y}\dfrac{dx}{dt}$ (d) $\dfrac{dx}{dt} = 2y\cos\left(y^2\right)\dfrac{dy}{dt}$

(e) $\dfrac{dy}{dt} = \sec\left(y^2\right)\dfrac{dx}{dt}$

21. _____ Which of the following is an antiderviative for $f(t) = \ln(t)$?

(a) $F(t) = 8 + \dfrac{1}{t}$ (b) $F(t) = t\ln(t) - t + 10$

(c) $F(t) = \ln(t) + t$ (d) $F(t) = e^t - 6$

(e) $F(t) = \dfrac{\ln^2(t)}{2}$

22. _____ Which of the following functions serves as an antiderivative for $f(x) = 2x\cos(x^2)$?

(a) $f(x) = 2\cos(x^2) - 4x^2\sin(x^2)$ (b) $f(x) = -2\sin(x^2)$

(c) $f(x) = \sin(x^2) - 8x + 5$ (d) $f(x) = \sin(x^2)$

(e) $f(x) = 4x^2\sin(x^2) - 8x$

23. _____ What are the input values where the tangent line to the graph of $f(x) = x + x^{2/3} - 8$ is undefined?

(a) There are no such input values. (b) Both $x = 1$ and $x = -1$

(c) Only $x = 0$ (d) Both $x = 0$ and $x = -8/27$

(e) Both $x = 1$ and $x = 1/2$

24. _____ What is the exact value of $\displaystyle\int_{-1}^{1}(x^2 + 2x)dx$?

(a) 2/3 (b) 2

(c) 4 (d) 0

(e) 3/2

25. _____ What is the value of $\displaystyle\lim_{x\longrightarrow 1}\frac{x^2 - 3x + 2}{x^2 - 4x + 3}$?

(a) 1 (b) 0

(c) 3/4 (d) 1/2

(e) Does not exist

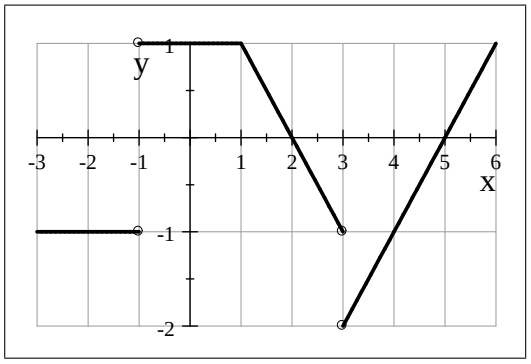
26. _____ The second derivative of $f(x) = e^{-x^2}$ is

(a) $f''(x) = -2xe^{-x^2}$ (b) $f''(x) = 2e^{-x^2}(2x^2 + 1)$

(c) $f''(x) = 4e^{-x^2}$ (d) $f''(x) == xe^{-x^2}(2 + x)$

(e) $f''(x) = e^{-x^2}$

Problems 27-31 refer to the graph of the function f shown below.



27. _____ According to the graph, $\lim_{x \longrightarrow 3^-} f(x)$

(a) is equal to -3 (b) is equal to -1

(c) is equal to -2 (d) is equal to -2.5

(e) Does not exist

28. _____ According to the graph, $\lim_{x \longrightarrow -1} f(x)$

(a) is equal to 1 (b) is equal to -1

(c) is equal to 0 (d) is equal to -2

(e) Does not exist

29. _____ The exact value of $\int_{-1}^{3} f(x)dx$ is

(a) 1/2 (b) 4

(c) 2 (d) 0

(e) 3

30. _____ Suppose that F is an antiderivative for f. At which input values will F have a local minimum output?

(a) At $x = 1$ (b) At $x = 2$

(c) At $x = 1$ and $x = 3$ (d) At $x = 2$ and $x = 5$

(e) At $x = -1$ and $x = 5$

31. _____ Suppose that F is an antiderivative for f. At which input values will F *not* be differentiable?

(a) At $x = 1$ only (b) At $x = -1$ only

(c) At $x = 3$ only (d) At $x = -1$, $x = 1$, and $x = 3$

(e) At $x = -1$ and $x = 3$

32. _____ The net area between the graph of $f(x) = 4x^3 - 1$ and the x-axis on the interval $0 \leq x \leq 2$ is

(a) 12 (b) 3.5
(c) 14 (d) 7
(e) 0

33. _____ The slope of the tangent line to the curve $\sqrt{x} - x\sqrt{y} = -1$ at the point $(1, 4)$ is

(a) $m = -12$ (b) $m = -6$
(c) $m = 1/12$ (d) $m = 1/6$
(e) undefined

34. _____ What is the exact value of $\displaystyle\int_{-1}^{1} \frac{3x}{x^2+1} dx$?

(a) 0 (b) 1
(c) $\dfrac{3}{2}\ln(2)$ (d) $\dfrac{\pi}{2}$
(e) $\dfrac{3}{2}$

35. _____ Suppose that the derivative function for a function f is given by $f'(x) = 3x^2 - 3$. Which of the following statements is true about the function f?

I The function f has a horizontal tangent line at $x = \pm 1$.
II The function f has a local maximum output at $x = 1$.
III The function f has a local minimum output at $x = -1$.

(a) Only Statement I. (b) Only Statements I and II.
(c) Only Statements I and III. (d) All are true.
(e) None are true.

36. _____ Suppose that f is a twice-differentiable function. If $f'(3) = 0$ and $f''(3) = 10$, then which of the following statements is true?

I The function f has an inflection point at $x = 3$.
II The function f is concave up at $x = 3$.
III The function f has a local minimum output at $x = 3$.

(a) Only Statement I. (b) Only Statement II.
(c) Only Statements I and III. (d) Only Statements II and III.
(e) None are true.

37. _____ At which values of x is the tangent line to $f(x) = x^3 + 3x - 5$ have slope $m = 15$?

(a) $x = \pm 2$ (b) $x = \pm\sqrt{3}$
(c) $x = 0$ (d) $x = 5$
(e) $x = \sqrt{5}$

38. _____ At what input values in the interval $-3 \leq x \leq 2$ will the slope of the tangent line to the graph of $f(x) = 4 - 5x + x^2$ be equal to the average rate of change for the function f on this input interval?

(a) $x = -1/2$ and $x = 1/2$ (b) $x = -1/2$

(c) $x = -1$ and $x = 1$ (d) $x = 1$ and $x = 2$

(e) $x = -3$ and $x = 1$

39. _____ The antiderivative family for the function $f(x) = \dfrac{x^2}{\cos^2(x^3)}$ would be

(a) $\displaystyle\int \frac{x^2}{\cos^2(x^3)}dx = -\frac{x^3 \csc\left(x^3\right)}{9} + C$ (b) $\displaystyle\int \frac{x^2}{\cos^2(x^3)}dx = \ln\left|\cos\left(x^3\right)\right| + C$

(c) $\displaystyle\int \frac{x^2}{\cos^2(x^3)}dx = \frac{\ln\left|\cos\left(x^3\right)\right|}{6} + C$ (d) $\displaystyle\int \frac{x^2}{\cos^2(x^3)}dx = \frac{x^3 \sec\left(x^3\right)}{3} + C$

(e) $\displaystyle\int \frac{x^2}{\cos^2(x^3)}dx = \frac{\tan\left(x^3\right)}{3} + C$

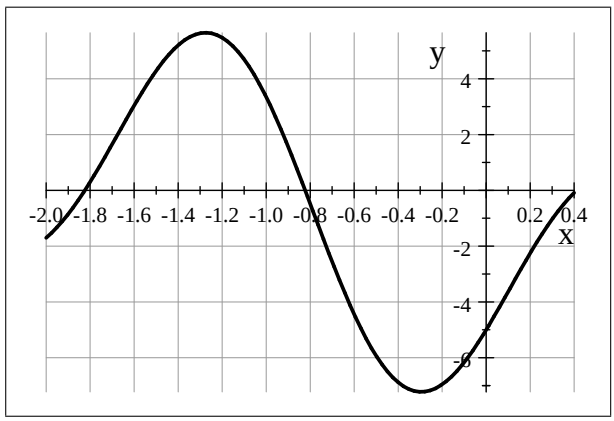
40. _____ The exact value of $\displaystyle\int_0^1 \frac{x^3+1}{\left(x^4+4x+1\right)^2}dx$ is

(a) $\dfrac{\ln(6)}{4}$ (b) $\dfrac{5}{24}$

(c) $\dfrac{4}{5}$ (d) $-\dfrac{4}{5}$

(e) $-\ln(6)$

Problems 41-45 refer to the graph of the function f shown below.



41. _____ Based on the graph above, the input values in the interval $-2 < x < 0.4$ guaranteed by the Mean Value Theorem occur at

(a) $x \approx -1.83$ and $x \approx -.83$ (b) $x \approx -1.83$ and $x \approx .37$

(c) $x \approx -1.35$ and $x \approx -.25$ (d) $x \approx -1.60$ and $x \approx .10$

(e) $x \approx -1.22$ and $x \approx -.40$

42. _____ Using a midpoint approximation with five rectangles, we find that $\int_{-1.8}^{-0.8} f(x)dx$ is approximately equal to

(a) 0 (b) 15.5

(c) -4.2 (d) 3.1

(e) 7.6

43. _____ At which of the following input values is $f'(x)$ clearly positive?

(a) $x = -1$ (b) $x = .1$

(c) $x = -.6$ (d) $x = -.3$

(e) $x = -.8$

44. _____ At which of the following input values is $f''(x)$ clearly positive?

(a) $x = -1.3$ (b) $x = -1$

(c) $x = .3$ (d) $x = -.3$

(e) $x = -.8$

45. _____ If F is an antiderivative for f, then we know that F has inflection points at

(a) $x \approx -1.83$, $x \approx -.83$, and $x \approx .10$ (b) $x \approx -1.83$, $x \approx -.83$, and $x \approx .10$

(c) $x \approx -1.35$ and $x \approx -.25$ (d) $x \approx -1.60$ and $x \approx .10$

(e) $x \approx -1.30$ and $x \approx -.30$

Problems 46-48 refer to the first and second derivative of a function f that are given below.

$$f'(x) = \frac{8x-1}{4x^2-x+1} \qquad f''(x) = -\frac{48x^2-8x+1}{(4x^2-x+1)^2}$$

46. _____ The function f has a horizontal tangent line at the input values

(a) $x = 1/8$ (b) $x = \dfrac{1 \pm \sqrt{15}}{8}$

(c) $x = 1/8$ and $x = \dfrac{1 \pm \sqrt{15}}{8}$ (d) $x = 0$

(e) The function f has no horizontal tangent lines.

47. _____ The function f has inflection points at the input values

(a) $x = \dfrac{1 \pm 4\sqrt{2}}{12}$ (b) $x = \dfrac{1 \pm \sqrt{15}}{8}$

(c) $x = \dfrac{1 \pm 4\sqrt{2}}{12}$ and $x = \dfrac{1 \pm \sqrt{15}}{8}$ (d) $x = 1/8$

(e) There are no inflection points.

48. _____ If we know that $f(0) = 2$, then we also know

(a) $f(x) = 2 + \ln\left|4x^2 - x + 1\right|$

(b) $f(x) = \dfrac{2}{4x^2 - x + 1}$

(c) $f(x) = \dfrac{\ln\left|4x^2 - x + 1\right|}{8} + 2$

(d) $f(x) = -\dfrac{1}{4x^2 - x + 1} + 3$

(e) $f(x) = \dfrac{2}{(4x^2 - x + 1)^2}$

49. _____ If we differentiate the curve $xy = \tan(y)$ with respect to x, we obtain the formula

(a) $\dfrac{dy}{dx} = \sec^2(y) - x$

(b) $\dfrac{dy}{dx} = \sec^2(y) + x$

(c) $\dfrac{dy}{dx} = \dfrac{y}{\sec^2(y) - x}$

(d) $\dfrac{dy}{dx} = \sec^2(y) + y$

(e) $\dfrac{dy}{dx} = x\sec^2(y)$

50. _____ A wall rises vertically from a patch of level ground, and a ten-foot ladder is leaning against this wall. If the top of the ladder is sliding down the wall at a constant speed of k feet per second, how fast is the base of the ladder moving when the top is b feet from the ground?

(a) $\dfrac{dx}{dt} = \dfrac{1}{\sqrt{100 - b^2}}(50 - bk)$ feet per second

(b) $\dfrac{dx}{dt} = \dfrac{50 - b}{\sqrt{100 - b^2}}k$ feet per second

(c) $\dfrac{dx}{dt} = -\dfrac{bk}{\sqrt{100 - b^2}}$ feet per second

(d) $\dfrac{dx}{dt} = b\left(50 - \sqrt{100 - b^2}k\right)$ feet per second

(e) $\dfrac{dx}{dt} = \dfrac{\sqrt{100 - b^2}}{b}k$ feet per second

ANSWERS

1) A	11) C	21) B	31) E	41) C
2) D	12) C	22) D	32) C	42) D
3) D	13) E	23) C	33) B	43) B
4) A	14) C	24) A	34) A	44) D
5) E	15) B	25) D	35) A	45) E
6) A	16) D	26) B	36) D	46) A
7) C	17) E	27) B	37) A	47) E
8) B	18) A	28) E	38) B	48) A
9) D	19) D	29) C	39) E	49) C
10) B	20) D	30) E	40) B	50) C