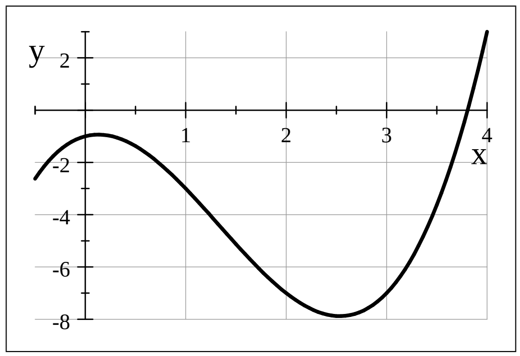
MATH 1910 PRACTICE EXAM III

1. ____________ Suppose that a function f is increasing and continuous on the half-open interval $-2 < x \leq 1$. Which of the following statements must be true?

   (a) The function f has no absolute extrema.
   (b) The function f has an absolute minimum at $x = 1$ and an absolute maximum at $x = -2$.
   (c) The function f has an absolute minimum at $x = -2$ and an absolute maximum at $x = 1$.
   (d) The function f has an absolute maximum at $x = 1$ and no absolute minimum.
   (e) The function f has an absolute minimum at $x = -2$ and no absolute maximum.

2. ____________ Suppose a function f is defined on the segment $-0.5 < x < 4$ by the graph shown below. Which of the following statements must be true for the function on this interval?



   (a) The function f has no absolute extrema.
   (b) The function f has an absolute maximum and an absolute minimum.
   (c) The function f has an absolute minimum but no absolute maximum.
   (d) The function f has an absolute maximum but no absolute minimum.
   (e) The function f has two absolute maxima but no absolute minima.

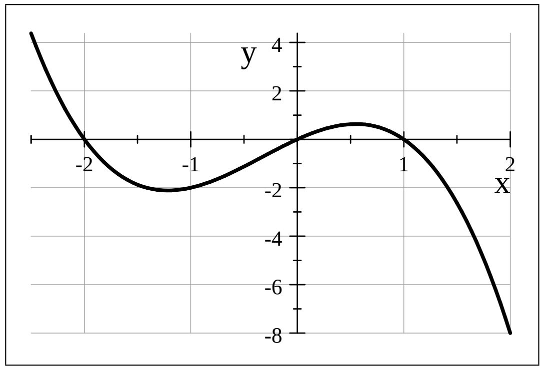
3. ____________ Suppose that f has a single critical number at $x = 3$. If $f'(0) = -2$ and $f'(4) = 6$, then the first derivative test tells us

   (a) f has no relative extrema.
   (b) f has a relative minimum output at $x = 3$.
   (c) f has a relative maximum output at $x = 3$.
   (d) f has an inflection point at $x = 3$
   (e) f has a horizontal tangent line at $x = 3$.

4. ____________ Suppose that $f\,'(2) = 0$. If $f''(2) = -5$, then we know

   (a) f has no relative extremum at $x = 2$.
   (b) f has a relative minimum output at $x = 2$.
   (c) f has a relative maximum output at $x = 2$.
   (d) f has an inflection point at $x = 2$.
   (e) f has a kink in its graph at $x = 2$.

The diagram below shows the *derivative* graph for a function f. Use this graph to answer Problems 5, 6, and 7.



5. ____________ Based on the *derivative* graph shown above, we see that f has critical numbers at

(a) $x = -2$, $x = 0$, and $x = 1$. (b) $x = -1.25$ and $x = .6$.

(c) only $x = .6$. (d) only $x = -1.25$.

(e) $x = -2.5$ and $x = 2$.

6. ____________ Based on the *derivative* graph shown above, we see that f has relative maximum outputs

(a) at $x = -2$ and $x = 1$. (b) only at $x = 0$.

(c) only at $x = .6$. (d) only at $x = -1.25$.

(e) at $x = -2.5$ and $x = 2$.

7. ____________ Based on the *derivative* graph shown above, we know that f will have inflection points at

(a) $x = -2$, $x = 0$, and $x = 1$. (b) $x = -1.25$ and $x = .6$.

(c) only $x = .6$. (d) only $x = -1.25$.

(e) $x = -2.5$ and $x = 2$.

Problems 8 - 10 refer to the function and its derivatives shown below.

$$f(x) = \ln\left(2x^2 - 2x + 1\right) \qquad f'(x) = \frac{2(2x-1)}{2x^2-2x+1} \qquad f''(x) = \frac{8x(1-x)}{\left(2x^2-2x+1\right)^2}$$

8. ____________ The function f will have critical numbers

(a) only at $x = 1/2$. (b) at $x = 0$ and $x = 1$.

(c) at $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$. (d) at $x = 1/2$ and $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$.

(e) at no value of x.

9. ____________ The function f will have a relative maximum output

(a) only at $x = 1/2$. (b) at $x = 0$ and $x = 1$.

(c) at $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$. (d) at $x = 1/2$ and $x = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$.

(e) at no value of x.

10. ____________ On the interval $[1,3]$, the function f will have its absolute minimum output at

(a) $x = 3$ (b) $x = \frac{1}{2}\left(1+\sqrt{3}\right)$

(c) $x = 1$ (d) $x = 1/2$

(e) $x = 2$

11. Consider the function $f(x) = x + \dfrac{1}{x}$.

(a) Compute the first derivative for f.

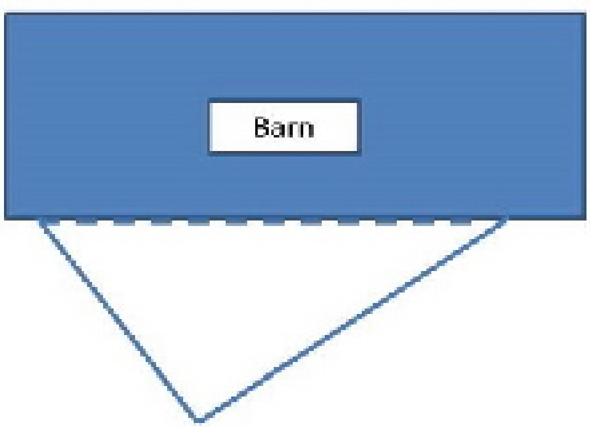
(b) Identify the critical numbers for f. Show your work.

(c) Use the First Derivative Test to determine which of these critical numbers yield relative maximum or minimum outputs for f. Show your steps.

12. What is the solution to the differential equation $y'' = x + \cos(x)$ if we also require $y'(0) = -1$ and $y(0) = 3$? You must show your steps for full credit.

13. Find the antiderivative family for the function $f(x) = 3x^{1/2} - \dfrac{4}{x^4} - 10\sec^2(x)$. You must show your work for full credit.

14. The area of a right triangle is given by $A = \frac{1}{2}xy$, where x and y represent the lengths of the legs of the triangle. A rancher wants to make a corral in the form of a right triangle adjacent to her barn. The barn will serve as the hypotenuse of the triangle and therefore that side of the corral needs no fencing. If the amount of fencing she has is fixed at 100 feet, and she wants to use all of the fencing, what should the lengths of the legs be to maximize the area of the corral?



15. Suppose instead that the rancher wants to orient the corral as shown in the figure below. (One leg is against the barn instead of the hypotenuse.) If she has 100 feet of fencing and wants to use all of it in making the corral, what should the lengths of the legs be to maximize the area of the corral?

HINT: If we let x represent the length in feet of the leg that needs fencing and let y represent the length in feet of the leg that does not need fencing, then the relationship between x and y is

$$100 = x + \sqrt{x^2 + y^2}$$