NAME:

10 pts 1. Let  $f(x) = \frac{x^2 - 5x + 6}{x^2 - x - 6}$ .

(a) The function f has a removable discontinuity at x = 3. Compute  $\lim_{x \to 3} f(x)$ . You must show your work and use proper limit notation for full credit.

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{x^2 - x - 6} = \lim_{x \to 3} \frac{(x - 3)(x - 2)}{(x - 3)(x + 2)} = \lim_{x \to 3} \frac{x - 2}{x + 2} = \frac{3 - 2}{3 + 2} = \frac{1}{5}$$

4 pts 2. Does  $\lim_{x \to 4} \frac{4x-3}{x^2-4}$  exist? Explain your answer.

If we input the value x = 4 into the denominator, we DO NOT get output 0. Therefore, the rational function  $f(x) = \frac{4x-3}{x^2-4}$  is continuous at x = 4. This tells us that the limit in question exists. In fact, the direct substitution principle tells us

$$\lim_{x \to 4} \frac{4x - 3}{x^2 - 4} = \frac{4(4) - 3}{4^2 - 4} = \frac{13}{12}$$

- 4 pts 3. Use the graph of the function f below to decide whether each limit exists. If it does, find the value; if not, explain why.
  - (a)  $\lim_{x \to 2^{+}} f(x) = 2$  (b)  $\lim_{x \to 4^{-}} f(x) \approx 4.5$  (c)  $\lim_{x \to 4^{+}} f(x) = 3$

(d) 
$$\lim_{x \to 3} f(x) = 3$$
 (e)  $\lim_{x \to -1^+} f(x) = 5$  (f)  $\lim_{x \to 4} f(x)$  DNE



The limit in Part (f) does not exist because there is a jump discontinuity in the graph of f at the input value x = 4. As a result, the values of the left-hand limit does not agree with the value of the right-hand limit.