MATH 1910 QUIZ 2

20 points

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6 pts 1. What is the formula for $\frac{df}{dt}$ if $f(t) = t^2\cos(t)$? You must show your steps for full credit.

$$\begin{aligned} \frac{df}{dt} &= \frac{d}{dt}\left[t^2\cos(t)\right] \\ &= \frac{d}{dt}\left[t^2\right]\cos(t) + t^2\frac{d}{dt}\left[\cos(t)\right] = 2t\cos(t) - t^2\sin(t) \end{aligned}$$

8 pts 2. What is the equation of the tangent line to the graph of $f(x) = x^{-1} + 2x^{-2}$ at the point $(1, f(1))$? You must show your steps for full credit.

Solution. We know that the equation of the tangent line will be $y = f'(1)\,[x-1] + f\,(1)$. Now,

$$f(1) = (1)^{-1} + 2(1)^{-2} = 1 + 2 = 3$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[x^{-1} + 2x^{-2}\right] \\ &= \frac{d}{dx}\left[x^{-1}\right] + 2\frac{d}{dx}\left[x^{-2}\right] \\ &= -x^{-2} - 4x^{-3} \end{aligned}$$

Therefore, we see that $f'(1) = -(1)^{-2} - 4(1)^{-3} = -5$. Consequently, the equation of the tangent line to the graph of f at the prescribed point will be

$$y = -5\,[x-1] + 3 \qquad \text{or} \qquad y = -5x + 8$$

6 pts 3. Differentiate the function $h(z) = \dfrac{1-z}{1+z}$. You must show your steps for full credit.

Solution. Let $f(z) = 1 - z$ and let $g(z) = 1 + z$. It follows that $f'(z) = -1$ and $g'(z) = 1$. Therefore, we know

$$\begin{aligned} \frac{dh}{dz} &= \frac{g(z)f'(z) - f(z)g'(z)}{g^2(z)} \\ &= \frac{(1+z)(-1) - (1-z)(1)}{(1+z)^2} \\ &= \frac{-1-z-1+z}{(1+z)^2} \\ &= -\frac{2}{(1+z)^2} \end{aligned}$$