MATH 1910 QUIZ 3

20 points

NAME:____________________________

In the problems that follow, let $f(x) = 2x^3 - 3x^2 - 12x + 1$. Note that Problem 3 appears on the back.

6 pts 1. Compute $f'(x)$ and use the derivative to identify the critical numbers for f. Do not classify the critical numbers. You must show your work for full credit.

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x+1)(x-2)$$

$$f'(x) = 0 \Longrightarrow 0 = 6(x+1)(x-2) \Longrightarrow x = -1 \quad \text{or} \quad x = 2$$

8 pts 2. Use the First Derivative Test to determine whether the critical numbers for f produce relative maximum or minimum outputs for f. You must show your work for full credit.

The two critical numbers partition the real number line into three sets — the ray $(-\infty, -1]$, the interval $[-1, 2]$, and the ray $[2, +\infty)$. Select a test value from each interval and check the sign of the first derivative output at each test value.

- In the ray $(-\infty, -1]$ let $x = -2$ and observe that $f'(-2) = 24 > 0$. This tells us that the graph of f is increasing on this ray.
- In the interval $[-1, 2]$ let $x = 0$ and observe that $f'(0) = -12 < 0$. This tells us that the graph of f is decreasing on this interval.
- In the ray $[2, +\infty)$ let $x = 3$ and observe that $f'(3) = 24 > 0$. This tells us that the graph of f is increasing on this ray.

Since the graph of f switches from increasing to decreasing at the input value $x = -1$, we may conclude that f has a relative maximum output when $x = -1$. Since the graph of f switches from decreasing to increasing at the input value $x = 2$, we may conclude that f has a relative minimum output when $x = 2$.

6 pts 3. Determine the values of x that produce the absolute maximum and absolute minimum output for f on the interval $0 \leq x \leq 3$. You must show your work for full credit.

First, note that the critical number $x = -1$ does not lie in the specified interval. Now, observe that

$$f(0) = 1 \qquad f(2) = -19 \qquad f(3) = -8$$

Comparing output, we see that on the specified interval, f has absolute maximum output of 1 when $x = 0$ and has absolute minimum output of -19 when $x = 2$.