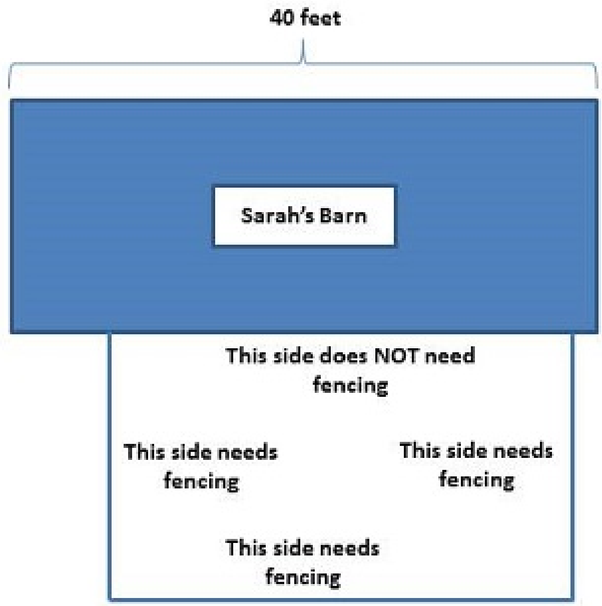
MATH 1910 QUIZ 4

20 points

NAME:__________________________

Sarah has 30 feet of chicken-wire fencing and wants to use all of the fencing to build a rectangular chicken pen against her barn as shown in the diagram below. The side of the pen against the 40-foot wall does not need any fencing, but the other sides do. What dimensions will maximize the area of the pen?



2 pts 1. Assign variable names to the changing quantities in this problem.

- Let A represent the area of the pen, measured in square feet.
- Let x represent the width of the pen, measured in feet.
- Let y represent the length of the pen, measured in feet.

2 pts 2. Construct the optimization formula for the problem. This formula must use all of the variables from Part (1).

$$A = xy$$

4 pts 3. What are the constraints in this problem?

Since x and y are dimensions, we must have $0 < x$ and $0 < y$. Since Sarah only has 100 feet of fencing, we know that the sum of the lengths of the three fenced sides must be 100 feet. This gives us the equation $2x + y = 100$ (assuming that y represents the side of the pen along the barn). Also, since the barn is 40 feet long, we know that $y \leq 40$. However, since Sarah only has 30 feet of fencing, this bound is irrelevant. (In fact, we know that we must have $y < 30$.)

2 pts 4. Use the constraints to rewrite the optimization formula as a function of only one variable.

We know that $y = 30 - 2x$, so we also know $A = f(x) = x(30 - 2x) = 30x - 2x^2$

2 pts 5. What is the relevant domain for the optimization function?

We know that we must have $0 < y$. Now, since we also know that $y = 30 - 2x$, it follows that we must have $x < 15$. Therefore, the relevant domain for this function is $0 < x < 15$.

4 pts 6. Determine the critical numbers for the optimization function.

$$f'(x) = 30 - 4x$$

$$f'(x) = 0 \Longrightarrow 30 - 4x = 0 \Longrightarrow x = 7.5 \text{ feet}$$

4 pts 7. Use either the First Derivative Test or the Second Derivative Test to determine which (if any) of the critical numbers in the relevant domain maximize the area.

$$f''(x) = -4 \Longrightarrow f''(7.5) < 0$$

Since we know that f has a critical number at $x = 7.5$, and since the graph of f is concave down when $x = 7.5$, we may conclude that the area of the pen is maximized when $x = 7.5$ feet. Note that $y = 15$ feet, and the maximum area is $A = 112.5$ square feet.