MATH 1910 EXAM II

100 points

NAME:____________________________

Write down the derivative function for each function below. These are worth five points each.

(1) $u(x) = \tan(x)$ $\qquad u'(x) = \sec^2(x)$

(2) $g(y) = \ln(y)$ $\qquad g'(y) = x^{-1}$

(3) $f(u) = u^{3/2}$ $\qquad f'(u) = \frac{3}{2}u^{1/2}$

(4) $k(a) = 4^a$ $\qquad k'(a) = 4^a \ln(4)$

(5) $j(b) = \arctan(b)$ $\qquad j'(b) = (1 + b^2)^{-1}$

Suppose f and g are differentiable functions. Match each derivative on the right with the general rule used to compute it. These are worth five points each.

(6) ___D___ $\frac{d}{dx}\left[f(x)g(x)\right] =$ $\qquad$ (A) $f'(x) + g'(x)$

(7) ___A___ $\frac{d}{dx}\left[f(x) + g(x)\right] =$ $\qquad$ (B) $\dfrac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

(8) ___B___ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ $\qquad$ (C) $g(x)f'(x) - f(x)g'(x)$

(D) $g(x)f'(x) + f(x)g'(x)$

(E) $f'(g(x))g'(x)$

5 pts **9.** ___B___ If we want to use the Chain Rule to differentiate $g(x) = (\tan(x))^{3/2}$, we know $\frac{dg}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$, where

(a) $f(u) = x$ and $u(v) = \tan^{3/2}(x)$ $\qquad$ (b) $u(x) = \tan(x)$ and $f(u) = u^{3/2}$

(c) $u(x) = x$ and $f(x) = \tan^{3/2}(x)$ $\qquad$ (d) $u(x) = x^{4/3}$ and $f(u) = \tan(u)$

(e) $u(x) = x$ and $f(u) = (\tan(u))^{3/2}$

5 pts **10.** ___A___ If we want to use the Product Rule to differentiate $g(x) = x\ln(x)$, then we know

(a) $\frac{dg}{dx} = x \cdot \frac{d}{dx}[\ln(x)] + \ln(x) \cdot \frac{d}{dx}[x]$ (b) $\frac{dg}{dx} = x \cdot \frac{d}{dx}[\ln(x)] - \ln(x) \cdot \frac{d}{dx}[x]$

(c) $\frac{dg}{dx} = \frac{d}{dx}[x] \cdot \frac{d}{dx}[\ln(x)]$ (d) $\frac{dg}{dx} = x \cdot \frac{d}{dx}[\ln(x)]$

(e) $\frac{dg}{dx} = \ln(x) \cdot \frac{d}{dx}[x]$

10 pts **11.** Find the derivative of $g(x) = (\tan(x))^{3/2}$. You must show your work for full credit.

$$\begin{aligned} \frac{dg}{dx} &= \frac{df}{du} \cdot \frac{du}{dx} \\ &= \frac{d}{du}\left[u^{3/2}\right] \frac{d}{dx}[\tan(x)] \\ &= \frac{3}{2}u^{1/2}\sec^2(x) \\ &= \frac{3}{2}\sqrt{\tan(x)}\sec^2(x) \end{aligned}$$

Let $u(x) = \tan(x)$ and $f(u) = u^{3/2}$

10 pts **12.** Find the derivative of $f(x) = x\ln(x) - x$. You must show your work for full credit.

$$\begin{aligned} \frac{df}{dx} &= \frac{d}{dx}[x\ln(x) - x] \\ &= \frac{d}{dx}[x\ln(x)] - \frac{d}{dx}[x] \\ &= x\frac{d}{dx}[\ln(x)] + \frac{d}{dx}[x]\ln(x) - \frac{d}{dx}[x] \\ &= x\left(\frac{1}{x}\right) + (1)\ln(x) - 1 \\ &= 1 + \ln(x) - 1 \\ &= \ln(x) \end{aligned}$$

10 pts **13.** Find the derivative of $f(x) = \dfrac{2-x}{3x}$. You must show your work for full credit.

$$\begin{aligned} \frac{df}{dx} &= \left(\frac{1}{(3x)^2}\right)\left(3x\frac{d}{dx}[2-x] - (2-x)\frac{d}{dx}[3x]\right) \\ &= \frac{(3x)(-1) - (2-x)(3)}{9x^2} \\ &= \frac{-3x - 6 + 3x}{9x^2} \\ &= -\frac{2}{3x^2} \end{aligned}$$

10 pts **14.** Find the equation of the tangent line to the graph of $f(x) = \dfrac{2-x}{3x}$ at the point $(3, f(3))$. You may use Problem 13. You must show your work for full credit.

Solution. The equation of the tangent line will be $y = f'(3)[x-3] + f(3)$. Now,

$$f(1) = \frac{2-3}{3(3)} = -\frac{1}{9} \qquad \text{and} \qquad f'(1) = -\frac{2}{3(3)^2} = -\frac{1}{27}$$

Therefore, the tangent line is $y = -\frac{1}{9}[x-3] - \frac{1}{27}$.

10 pts **15.** Find a formula for $\dfrac{dy}{dx}$ if $y^3 - 2y = \cos(x)$. You must show your work for full credit.

$$\begin{aligned}
y^3 - y = x^3 \quad \implies \quad & \frac{d}{dx}\left[y^3 - 2y\right] = \frac{d}{dx}\left[\cos(x)\right] \\
\implies \quad & \frac{d}{dx}\left[y^3\right] - 2\frac{d}{dx}\left[y\right] = \frac{d}{dx}\left[\cos(x)\right] \\
\implies \quad & 3y^2\frac{dy}{dx} - 2\frac{dy}{dx} = -\sin(x) \\
\implies \quad & \left(3y^2 - 2\right)\frac{dy}{dx} = -\sin(x) \\
\implies \quad & \frac{dy}{dx} = -\frac{\sin(x)}{3y^2 - 2}
\end{aligned}$$