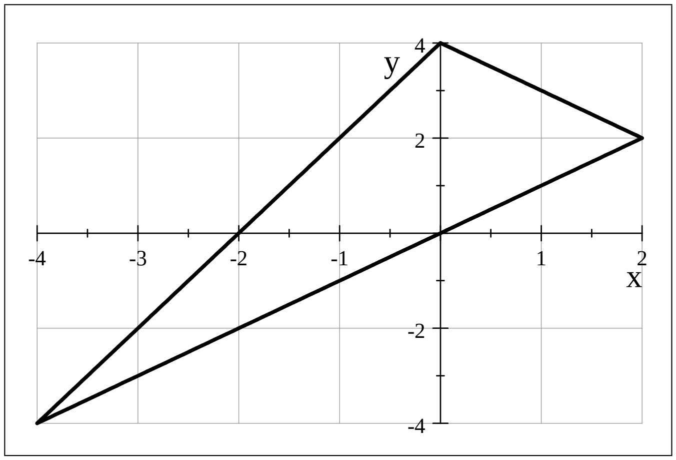
MATH 1920 PRACTICE EXAM II

1. Let $\mathcal{R}$ be the finite region enclosed between the straight lines $f(x) = 4 - x$, $g(x) = x$ and $h(x) = 4 + 2x$



(a) Set up the definite integral that computes the area of $\mathcal{R}$.

$$A = \int_{-4}^{0} [4 + x]\, dx + \int_{0}^{2} [4 - 2x]\, dx$$

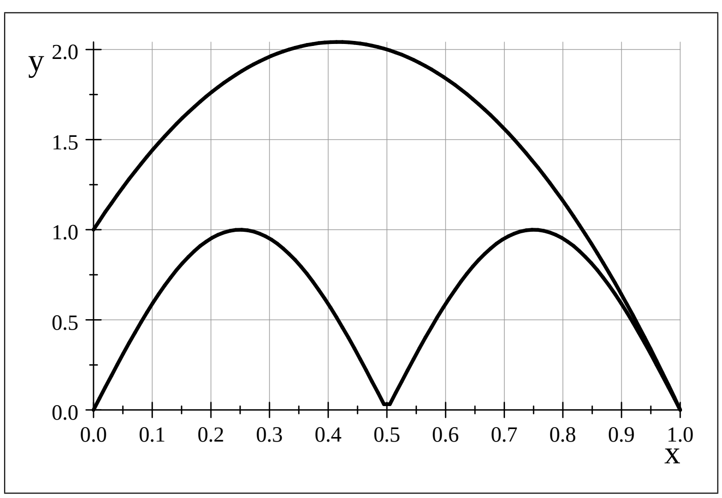
(b) Let V be the volume obtained by revolving $\mathcal{R}$ about the line $x = 2$. Set up the definite integral that computes V using the Method of Washers.

Solution. When using the Method of Washers, cross sections must be taken perpendicular to the axis of revolution. This means we must recast the bounding curves as functions of y. Now

$$y = f(x) = 4 - x \Longrightarrow x = f^{-1}(y) = 4 - y \qquad y = h(x) = 4 + 2x \Longrightarrow x = h^{-1}(y) = \frac{1}{2}(y - 4)$$

$$V = \pi \int_{-4}^{2} \left[\left(\frac{y}{2} - 4 \right)^2 - (y - 2)^2 \right] dy + \pi \int_{2}^{4} \left[\left(\frac{y}{2} - 4 \right)^2 - (2 - y)^2 \right] dy$$

2. Let $\mathcal{R}$ be the region enclosed between the curves $f(x) = -6x^2 + 5x + 1$ and $g(x) = |\sin(2\pi x)|$ on the interval $0 \le x \le 1$.



(a) Set up the definite integral that computes the area of $\mathcal{R}$.

$$A = \int_0^1 \left[1 + 5x - 6x^2 - |\sin(2\pi x)|\right] dx$$

(b) Let V be the volume obtained by revolving $\mathcal{R}$ about the x-axis. Set up the definite integral that computes V using the Method of Washers.

$$V = \pi \int_0^1 \left[\left(1 + 5x - 6x^2\right)^2 - \sin^2(2\pi x)\right] dx$$

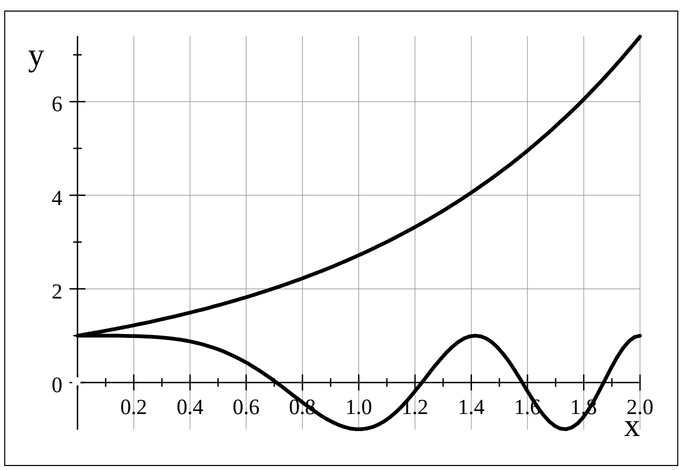
3. Use integraton by parts to compute $\displaystyle\int_0^2 xe^x dx$.

$$\begin{aligned} \int_0^2 xe^x dx &= xe^x|_0^2 - \int_0^2 e^x dx \qquad \text{Let } u = x \text{ and } \frac{dv}{dx} = e^x \\ &= xe^x|_0^2 - e^x|_0^2 \\ &= 2e^2 - (e - 1) \\ &= e^2 + 1 \end{aligned}$$

4. Use an appropriate substitution to help compute $\displaystyle\int_0^2 x\cos\left(\pi x^2\right) dx$.

$$\begin{aligned} \int_0^2 x\cos\left(\pi x^2\right) dx &= \frac{1}{2\pi}\int_{u=0}^{u=4\pi} \cos(u)du \qquad \text{Let } u = \pi x^2 \text{ so } \frac{du}{dx} = 2\pi x \\ &= \frac{1}{2\pi}\sin(u)\Big|_0^{4\pi} \\ &= 0 \end{aligned}$$

5. Let $\mathcal{R}$ be the region enclosed between the curves $f(x) = e^x$ and $g(x) = \cos\left(\pi x^2\right)$ on the interval $0 \le x \le 2$.



Let V be the volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis. Use the Method of Shells to compute V.

$$\begin{aligned} V &= 2\pi \int_0^2 x\left(e^x - \cos(\pi x^2)\right)dx \\ &= 2\pi \int_0^2 xe^x\,dx - 2\pi \int x\cos(\pi x^2)dx \\ &= 2\pi\left(e^2+1\right) \end{aligned}$$

6. Let $\mathcal{R}$ be the region in Problem 5, and let V be the volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = 2$. Set up the definite integral the computes V using the Method of Shells.

$$V = 2\pi \int_0^2 (2-x)\left(e^x - \cos(\pi x^2)\right)dx$$