MATH 1920 PRACTICE EXAM I

100 points

NAME:____________________________

Please place the letter of your selection in the blank provided. These questions are worth five points each.

5 pts 1. ____________One antiderivative for $f(x) = e^x$ is the function

(a) $F(x) = \frac{1}{2}e^{x^2}$ (b) $F(x) = e^x - \sqrt{2}$

(c) $F(x) = \frac{1}{2}e^{2x}$ (d) $F(x) = \ln(x) + \pi$

(e) $F(x) = 3e^x$

5 pts 2. ____________ In order to compute the antiderivative family for $f(x) = \frac{\cos(1/x)}{x^2}$ we need the substitution

(a) $u = \cos(x)$ (b) $u = \frac{1}{x}$

(c) $u = \frac{1}{x^2}$ (d) $u = x^2$

(e) $u = x$

5 pts 3. ____________ The function $F(x) = x\sin(x) + 10$ is a solution to which of the following differential equations?

(a) $y' = x\cos(x) + \sin(x)$ (b) $y' = \frac{x^2}{2}\sin(x) - \cos(x)$

(c) $y' = -\frac{x^2}{2}\cos(x)$ (d) $y' = x\cos(x)$

(e) $y' = \frac{x^2}{2}\cos(x)$

In Problem 4, suppose we know that

$$\int_a^b f(x)dx = -5 \qquad \int_a^b g(x)dx = 10$$

5 pts 4. ____________ Based on this information, we know $\int_b^a [3f(x) - 6g(x)]\,dx$

(a) is equal to 75 (b) is equal to -5

(c) is equal to 5 (d) is equal to -45

(e) is equal to -75

5 pts 5. ____________ By making an appropriate substitution, we know that $\int \frac{x^2}{(1+x^3)^4}dx$

(a) is equal to $3\int \frac{u}{(1+u)}du$ (b) is equal to $\int x^2\left(\frac{1}{u^4}\right)du$

(c) is equal to $\frac{1}{2}\int \frac{u}{1+u^6}du$ (d) is equal to $\int u^{-4}du$

(e) is equal to $\frac{1}{3}\int u^{-4}du$

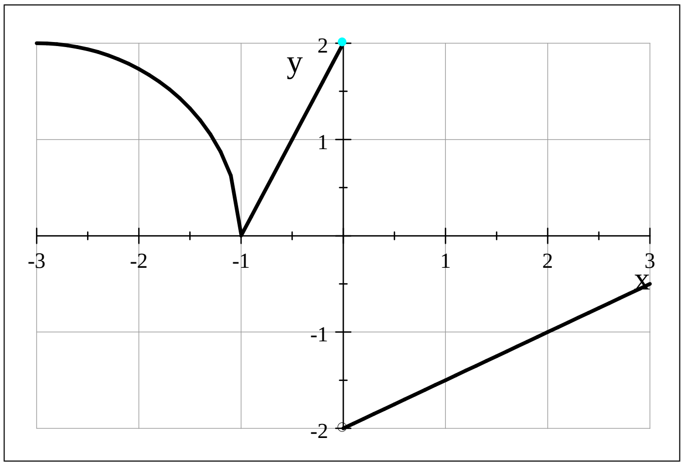
5 pts 6. ____________ After applying the appropriate substitution in $\int_{x=1}^{x=4} \frac{\sin(2+\ln(x))}{x} dx$, the new limits will be

(a) $u = 1$ and 4 (b) $u = 1$ and $u = 1/4$
(c) $u = 2$ and $u = 2 + \ln(4)$ (d) $u = \sin 2$ and $u = \sin(2 + \ln(4))$
(e) $u = 1$ and $u = 2$

5 pts 7. ____________ If $F(t) = \int_1^t x\sqrt{x^2-1}dx$, then $F'(3)$

(a) is equal to $\sqrt{3}$ (b) is equal to $2\sqrt{2}$
(c) is equal to $\frac{2\sqrt{2}}{3}$ (d) is equal to $\frac{2\sqrt{3}}{3}$
(e) is equal to $6\sqrt{2}$

Problems 8 - 10 refer to the graph of a function f below. The curve is one-quarter of a circle of radius 2.



5 pts 8. ____________ Based on this diagram, we know $\int_{-3}^{2} f(x)dx$

(a) is equal to $\frac{4\pi - 7}{4}$ (b) is equal to $\pi - 1$
(c) is equal to $\frac{\pi}{4} - 3$ (d) is equal to $\pi - 2$
(e) is equal to $3 - \frac{\pi}{4}$

5 pts 9. ____________ If $F(x) = \int_0^x f(t)dt$, then $F(-1)$

(a) is equal to 1 (b) is equal to 0
(c) is equal to $\frac{1}{2}$ (d) is equal to $-\frac{1}{2}$
(e) is equal to -1

5 pts 10. ____________ The arc-lenth of the function $f(x) = \cos^2(x)$ on the interval $0 \le x \le \pi$ is given by the formula

(a) $\displaystyle\int_0^\pi \cos(x)dx$

(b) $\displaystyle\int_0^\pi \cos^2(x)dx$

(c) $\displaystyle\int_0^\pi (1 + 2\sin(x)\cos(x))\, dx$

(d) $\displaystyle\int_0^\pi \sqrt{1 + 2\sin(x)\cos(x)}dx$

(e) $\displaystyle\int_0^\pi \sqrt{1 + 4\sin^2(x)\cos^2(x)}dx$

10 pts 11. What is the average value of the function $f(x) = x - \dfrac{1}{x}$ on the interval $1 \le x \le 6$? Show your work.

15 pts 12. Use the Fundamental Theorems of Calculus to compute $\displaystyle\int_{-1}^{3} \frac{x^2}{(1+x^3)^4} dx$. Show your work.

15 pts 13. Consider the function $f(x) = \sqrt{1 + 4x^2}$ on the interval $[1, 3]$.

(a) If we divide $[1, 3]$ into six subintervals of equal width, then the partition we obtain is

$x_0 =$ ____________ $x_1 =$ ____________ $x_2 =$ ____________ $x_3 =$ ____________

$x_4 =$ ____________ $x_5 =$ ____________ $x_6 =$ ____________

(b) Use a trapezoid estimate with six subintervals to estimate $\int_1^3 f(x)dx$. Show your work.

10 pts 14. Use the Fundamental Theorems of Calculus to compute the exact value of $\int_0^1 \left(2 + \sin(\pi x) - \frac{x}{1 + x^2}\right) dx$. Show your work.