Practice for Exam III

1. It can be shown that
$$f(x) = \frac{2+7x-3x^2}{x^3-x^2-x+1} = \frac{1}{x-1} + \frac{2}{x+1} - \frac{3}{(x-1)^2}$$
Use this fact to determine the antiderivative family for f.

2. It can be shown that
$$f(x) = \frac{8x^5+4x^4-14x^3-3x^2+10x}{8x^3+4x^2+2x+1} = x^2 - 2 + \frac{3}{2x+1} - \frac{4x+1}{1+4x^2}$$
Use this fact to determine the antiderivative family for f.

3. Construct the partial fraction decomposition for $f(x) = \dfrac{7x^2-4x+3}{x^3-x^2}$.

4. Find the antiderivative family for $g(\theta) = \dfrac{\sin^3(\theta)}{\cos^2(\theta)}$.

5. Use Problem 4 to help find the antiderivative family for $y = \dfrac{x^3}{(1-x^2)^{3/2}}$.

6. Using an appropriate substitution, it can be shown that
$$\int \frac{x^2-1}{\sqrt{1+4x^2}}dx = \frac{\sec\theta\tan\theta}{16} - \frac{9}{16}\ln|\sec\theta+\tan\theta| + C$$
   (a) What substitution was made?

   (b) Rewrite the antiderivative family as functions of x instead of functions of θ.

7. Using an appropriate substitution, it can be shown that
$$\int \frac{1}{\sqrt{x}(1+x)}dx = 2\arctan(u) + C$$
   (a) What substitution was made?

   (b) Use this fact to evaluate the improper integral $\displaystyle\int_0^{+\infty} \frac{1}{\sqrt{x}(1+x)}dx$.

8. Since $-1 \leq \cos(x) \leq 1$, it is true that
$$\frac{\sqrt{x}-1}{x^2} \leq \frac{\sqrt{x}+\cos(x)}{x^2} \leq \frac{\sqrt{x}+1}{x^2}$$
Use this fact and the Comparison Test to decide whether or not $\displaystyle\int_1^{+\infty} \frac{\sqrt{x}+\cos(x)}{x^2}dx$ converges.

ANSWERS

1. $\displaystyle\int \left(\frac{1}{x-1}+\frac{2}{x+1}-\frac{3}{(x-1)^2}\right)dx = \ln|x-1|+2\ln|x+1|+\frac{3}{x-1}+C$

2. $\displaystyle\int \left(x^2-2+\frac{3}{2x+1}-\frac{4x+1}{1+4x^2}\right)dx = \tfrac{x^3}{3}-2x+\tfrac{3}{2}\ln|2x+1|-\tfrac{1}{2}\left(\arctan(2x)-4\ln(1+4x^2)\right)+C$

3. $\displaystyle\frac{7x^2-4x+3}{x^3-x^2} = \frac{1}{x}-\frac{3}{x^2}+\frac{6}{x-1}$

4. $\displaystyle\int \frac{\sin^3(\theta)}{\cos^2(\theta)}d\theta = \cos(\theta)+\sec(\theta)+C$

5. $\displaystyle\int \frac{x^3}{(1-x^2)^{3/2}}dx = \frac{2-x^2}{\sqrt{1-x^2}}+C$

6. The substitution is $\tan(\theta)=2x$.

$$\int \frac{x^2-1}{\sqrt{1+4x^2}}dx = \frac{x\sqrt{1+4x^2}}{8}-\frac{9}{16}\ln\left|\sqrt{1+4x^2}+2x\right|+C$$

7. The substitution is $u=\sqrt{x}$.

$$\int_0^{+\infty}\frac{1}{\sqrt{x}(1+x)}dx = \lim_{a\longrightarrow 0^+}\int_a^1 \frac{1}{\sqrt{x}(1+x)}dx + \lim_{h\longrightarrow +\infty}\int_1^h \frac{1}{\sqrt{x}(1+x)}dx = \pi$$

8. The improper integral in question converges, since

$$\int_1^{+\infty}\frac{\sqrt{x}+1}{x^2}dx = 3$$