MATH 1920 EXAM I

100 points

NAME:____________________________

1. Write down the antiderivative family for each function below. These are worth two points each.

(a) $f(x) = \sin(x)$ $\quad\int f(x)dx = -\cos(x) + C$

(b) $g(y) = \sec^2(y)$ $\quad\int g(y)dy = \tan(y) + C$

(c) $h(z) = \dfrac{1}{z}$ $\quad\int h(z)dz = \ln|z| + C$

(d) $k(a) = e^a$ $\quad\int k(a)da = e^a + C$

(e) $j(b) = b^{1/2}$ $\quad\int j(b)db = \dfrac{2}{3}b\sqrt{b} + C$

Please place the letter of your selection in the blank provided. These questions are worth five points each.

5 pts 2. ___**C**___ One solution to the differential equation $y' = \sin(x) + 1$ will be

(a) $y = \cos(x)$ (b) $y = \cos\left(\dfrac{x^2}{2}\right) + 5$

(c) $y = -\cos(x) + x - \pi$ (d) $y = \sin(x) + x + 2$

(e) $y = \arcsin(x) - 3$

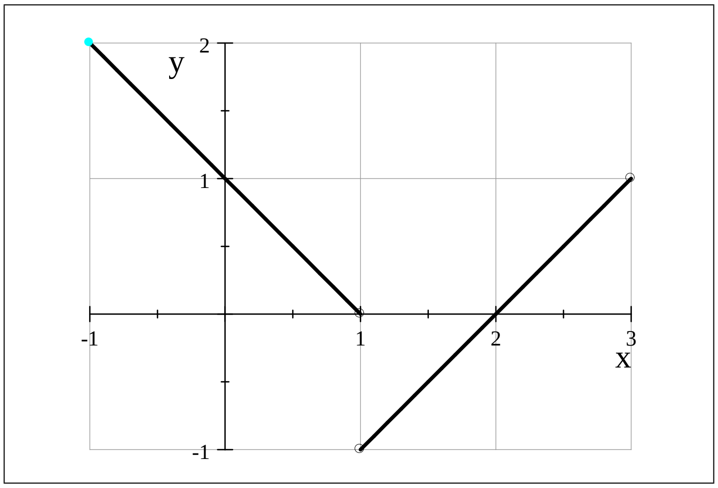
5 pts 3. ___**C**___ After making the appropriate u substitution for $\displaystyle\int_{t=3}^{t=5} \frac{2t}{\sqrt{t^2-1}}dt$ the new limits will be

(a) $u = 0$ and $u = 2$ (b) $u = 6$ and $u = 10$

(c) $u = 8$ and $u = 24$ (d) $u = 3$ and $u = 5$

(e) $u = 2\sqrt{2}$ and $u = 2\sqrt{6}$

Use the graph of a function f below to answer Problem 4.



5 pts 4. **D** Based on the graph shown above, we see that $\int_{-1}^{2} f(x)dx$ is equal to

(a) $1/2$ (b) 1

(c) -1 (d) $3/2$

(e) 0

5 pts 5. **E** If we partition the interval $[3, 13]$ into twelve subintervals of equal width, then each subinterval will have width

(a) $\triangle x = 10$ (b) $\triangle x = 1$

(c) $\triangle x = \frac{1}{2}$ (d) $\triangle x = 12$

(e) $\triangle x = \frac{5}{6}$

5 pts 6. **C** Which of the following formulas gives the arc-length for the graph of the function $f(x) = \ln(x)$ on the interval $1 \le x \le 4$?

(a) $\int_1^4 \left(1 + \frac{1}{x}\right) dx$ (b) $\int_1^4 \sqrt{1 + \frac{1}{x}} dx$

(c) $\int_1^4 \sqrt{1 + \frac{1}{x^2}} dx$ (d) $\int_1^4 \ln(x) dx$

(e) $\int_1^4 (x \ln(x) - x)\, dx$

5 pts 7. **A** Which of the following formulas gives the average value for $f(x) = \sqrt{x}$ on the interval $1 \le x \le 9$?

(a) $\frac{1}{8} \int_1^9 \sqrt{x} dx$ (b) $\int_1^9 \sqrt{x} dx$

(c) $\frac{1}{2} \int_1^3 \frac{1}{2\sqrt{x}} dx$ (d) $\frac{1}{8} \int_1^9 x dx$

(e) $\frac{1}{4} \int_1^3 \sqrt{1 + \frac{1}{4x}} dx$

10 pts 8. Solve the initial value problem $y' = 2x + x^{-1}$, where $y(1) = 2$. Show your work.

Solution. First, observe that the solution family for this differential equation will be

$$y = \int \left[2x + x^{-1}\right] dx = x^2 + \ln|x| + C$$

Now, under the assumption that $y(1) = 2$, we know that

$$2 = y(1) = 1^2 + \ln(1) + c \Longrightarrow 2 = 1 + C \Longrightarrow 1 = C$$

Therefore, the particular solution we want is $y = x^2 + \ln|x| + 1$.

15 pts 9. Find the average value of $f(x) = \sqrt{x}$ on the interval $1 \le x \le 9$. Show your work.

10.

$$A = \frac{1}{8}\int_1^9 x^{1/2}dx = \frac{1}{8}\left(\frac{2}{3}\right)x\sqrt{x}\Big|_1^9 = \frac{1}{12}\left[27 - 1\right] = \frac{13}{6}$$

15 pts 11. Find the exact value of $\displaystyle\int_0^{-1} \frac{1}{(4x-1)^2}dx$. Show your work.

Solution. To begin, let $u = 4x - 1$. This tells us that $\frac{du}{dx} = 4$. When $x = 0$, we know $u = -1$; and, when $x = -1$, we know $u = -5$. Therefore,

$$\begin{aligned}
\int_0^{-1} \frac{1}{(4x-1)^2}dx &= \int_{u=-1}^{u=-5} u^{-2}\left[\left(\frac{1}{4}\right)\frac{du}{dx}\right]dx \\
&= \frac{1}{4}\int_{u=-1}^{u=-5} u^{-2}du \\
&= -\frac{1}{4}\left(\frac{1}{u}\right)\Big|_{u=-1}^{u=-5} \\
&= -\frac{1}{4}\left[-\frac{1}{5} + 1\right] \\
&= -\frac{1}{5}
\end{aligned}$$

15 pts 12. Suppose you are given the following table of data for a function f and are asked to estimate the value of $\int_1^3 f(x)dx$.

Index j	0	1	2	3	4	5
Value of x_j	1.0	1.4	1.8	2.2	2.6	3.0
Approximate value of $f\ (x_j)$	−2.78	−1.87	0.52	0.78	−0.25	−1.15

(a) What is the width of each subinterval?

Based on the table, each subinterval has width $\triangle x = 0.4$. On the other hand, notice that there are five subintervals being used —

$$I_0 = [1, 1.4] \qquad I_1 = [1.4, 1.8] \qquad I_2 = [1.8, 2.2] \qquad I_3 = [2.2, 2.6] \qquad I_4 = [2.6, 3.0]$$

Therefore, the width of each subinterval will be $\triangle x = (3-1)/5 = 0.4$.

(b) Use the Trapezoid Rule to estimate $\displaystyle\int_1^3 f(x)dx$. Show your work.

$$\int_1^3 f(x)dx \approx \left(\frac{1}{2}\right)(0.4)\left[-2.78 + (2)(-1.87) + (2)(0.52) + (2)(0.78) + (2)(-0.25) - 1.15\right] \approx -1.11$$