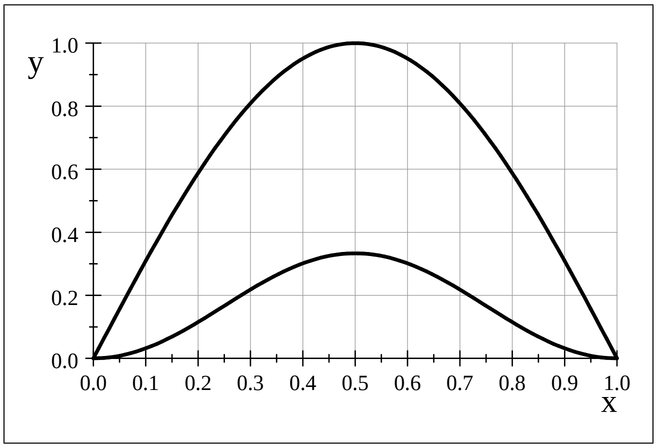
# MATH 1920 REVIEW FOR FINAL

The diagram below shows the region $\mathcal{R}$ enclosed between the curves $f(x) = \frac{1}{3}\sin^2(\pi x)$ and $g(x) = \sin(\pi x)$ on the interval $0 \le x \le 1$. Problems 1 - 3 refer to this diagram.




1. ____________The area of the region $\mathcal{R}$ is given by the formula

(a) $A = \int_0^1 \left[\frac{1}{3}\sin^2(\pi x) - \sin(\pi x)\right] dx$

(b) $A = \frac{1}{3}\int_0^1 \sin(\pi x)\left[\sin(\pi x) - 3\right] dx$

(c) $A = \frac{1}{3}\int_0^1 \left[3x\sin(\pi x) - x\sin^2(\pi x)\right] dx$

(d) $A = \frac{1}{\pi}\int_0^1 \left[\arcsin(\sqrt{3y}) - \arcsin(y)\right] dy$

(e) $A = \int_0^1 \left[\sin(\pi x) - \frac{1}{3}\sin^2(\pi x)\right] dx$

2. ____________If we revolve the region $\mathcal{R}$ about the $x$-axis, which of the following formulas gives the volume of the resulting solid?

(a) $V = \frac{\pi}{9}\int_0^1 \left[9\sin^2(\pi x) - \sin^4(\pi x)\right] dx$

(b) $V = 2\pi\int_0^1 x\left[\sin(\pi x) - \frac{1}{3}\sin^2(\pi x)\right] dx$

(c) $V = \pi\int_0^1 \left[\frac{1}{9}\sin^2(\pi x) - 1\right]\sin^2(\pi x) dx$

(d) $V = 2\int_0^1 y\left[\arcsin(y) - \arcsin(\sqrt{3y})\right] dy$

(e) $V = \int_0^1 \left[\arcsin^2(y) - \arcsin^2(\sqrt{3y})\right] dy$

3. ____________If we revolve the region $\mathcal{R}$ about the $y$-axis, which of the following formulas gives the volume of the resulting solid?

(a) $V = \frac{\pi}{9}\int_0^1 \left[9\sin^2(\pi x) - \sin^4(\pi x)\right] dx$

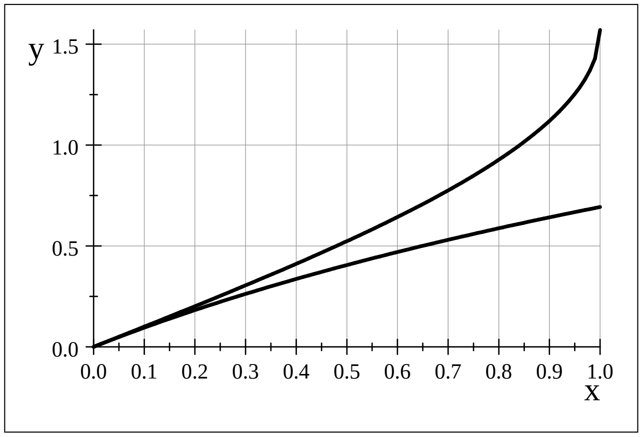
(b) $V = 2\pi\int_0^1 x\left[\sin(\pi x) - \frac{1}{3}\sin^2(\pi x)\right] dx$

(c) $V = \pi\int_0^1 \left[\frac{1}{9}\sin^2(\pi x) - 1\right]\sin^2(\pi x) dx$

(d) $V = 2\int_0^1 y\left[\arcsin(y) - \arcsin(\sqrt{3y})\right] dy$

(e) $V = \int_0^1 \left[\arcsin^2(y) - \arcsin^2(\sqrt{3y})\right] dy$

The diagram below shows the region $\mathcal{R}$ enclosed between the curves $f(x) = \arcsin(x)$ and $g(x) = \ln(x+1)$ on the interval $0 \le x \le 1$. Problems 4 and 5 refer to this diagram.




4. ____________ Which of the following formulas correctly gives the area of the region $\mathcal{R}$?

(a) $A = \int_0^1 \left[\ln^2(x+1) - \arcsin^2(x)\right] dx$

(b) $A = \int_0^1 \left[\ln(x+1) - \arcsin(x)\right] dx$

(c) $A = \int_0^1 \left[\arcsin^2(x) - \ln^2(x+1)\right] dx$

(d) $A = \int_0^{\ln(2)} \left[e^y - 1 - \sin(y)\right] dy + \int_{\ln(2)}^{\pi/2} \left[1 - \sin(y)\right] dy$

(e) $A = \int_0^{\pi/2} \left[\sin(y) - e^y + 1\right] dx$

5. ____________ If we revlove $\mathcal{R}$ about the line $y = 2$, which of the following formulas gives the volume of the resulting solid?

(a) $V = \pi \int_0^1 \left[2 - (\ln(x+1) - \arcsin(x))^2\right] dx$

(b) $V = 2\pi \int_0^1 y\left[2 - (e^y - 1 - \sin(y)\right] dy$

(c) $V = \pi \int_0^1 \left[(\ln(x+1) - 2)^2 - (\arcsin(x) - 2)^2\right] dx$

(d) $V = 2\pi \int_0^1 y\left[2 - (\sin(y) - e^y + 1)\right] dy$

(e) $V = \pi \int_0^1 \left[\ln^2(x+1) - \arcsin^2(x)\right] dx$

6. ____________ What is the particular solution to the initial value problem $y'' = \frac{1}{x} + 1$, if we assume $x > 0$, $y(1) = 2$ and $y'(1) = 0$?

(a) $y = \ln^2 \sqrt{x} - x + \frac{1}{2}$

(b) $y = x\ln(x) + \frac{1}{2}\left(x^2 - 1\right)$

(c) $y = x\ln(x) - 2x + \frac{1}{2}x^2$

(d) $y = \ln^2(x) + x - 3$

(e) $u = x - \ln(x) + x^2 + \frac{11}{3}$

7. ____________What can be said about the improper integral $\int_0^{+\infty} \frac{1}{(x+1)^{1/3}} dx$?

(a) The integral diverges.

(b) The integral converges to $\frac{3}{\sqrt[3]{2}}$.

(c) The integral converges to $\frac{1}{\sqrt[3]{2}}$.

(d) The integral converges to $\sqrt[3]{2}$.

(e) The integral converges to $\sqrt[3]{4}$.

8. ____________In order to use Integration by Parts to evaluate $\int \ln(x) dx$ we would let

(a) $u = \ln(x)$ and $dv = 1dx$

(b) $u = x$ and $dv = \ln(x)dx$

(c) $u = e^x$ and $dv = 1dx$

(d) $u = \ln(x)$ and $dv = xdx$

(e) $u = 1$ and $dv = e^x dx$

9. ____________What is the exact value of the series $\sum_{j=3}^{\infty} (-1)^j \frac{2}{3^{j+1}}$?

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{1}{27}$

(d) $-\frac{1}{18}$

(e) $-\frac{1}{54}$

10. ____________After making an appropriate substitution, what would the new limits be on the definite integral below?

$$\int_0^{1/2} \frac{x}{\sqrt{1-x^4}} dx$$

(a) $u = 0$ and $u = 1$

(b) $u = 0$ and $u = 1/16$

(c) $u = 1$ and $u = 15/16$

(d) $u = 1$ and $u = 3/4$

(e) $u = 0$ and $u = 1/4$

11. ____________In order to evaluate $\int \tan^3(\theta) \sec^2(\theta)\, d\theta$ it would be best to

(a) let $u = \sec^2(\theta)$

(b) observe that $\tan^3(\theta)\sec^2(\theta) = \sec^4(\theta) - \sec^2(\theta)$

(c) let $u = \sec(\theta)$

(d) observe that $\tan^3(\theta)\sec^2(\theta) = \tan^3(\theta) + \tan^5(\theta)$

(e) let $u = \tan(\theta)$

12. In order to evaluate $\int \sin^2(x)\cos^2(x)dx$ it would be best to

(a) let $u = \sin(x)$ (b) first apply the double angle formulas

(c) let $u = \cos^2(x)$ (d) first apply the Pythagorean Identity

(e) let $u = \sin^4(x)$

13. ____________Which of the following formulas gives the arc-length for the function $f(x) = e^{2x}$ on the interval $1 \le x \le 2$?

(a) $\mathcal{L} = \int_1^2 \sqrt{1+e^{2x}}dx$ (b) $\mathcal{L} = \int_1^2 \sqrt{1+\frac{1}{x^2}}dx$

(c) $\mathcal{L} = \int_1^2 \sqrt{1+4e^{4x}}dx$ (d) $\mathcal{L} = \frac{1}{2}\int_1^2 \sqrt{1+\ln^2(x)}dx$

(e) $\mathcal{L} = \frac{1}{2}\int_1^2 \frac{\sqrt{x^2+1}}{x}dx$

14. ____________In order to evaluate $\int \sin^5(y)\cos^2(y)dy$ it would be best to

(a) let $u = \sin(y)$ (b) first apply the double angle formulas

(c) let $u = \cos^2(y)$ (d) observe that $\sin^5(y)\cos^2(y) = \left[\cos^2(y) - \cos^4(y)\right]\sin(y)$

(e) let $u = \sin^5(y)$

15. ____________After performing a partial fraction decomposition, it can be shown that

$$\int \frac{2+3x^2-x^3}{x^4+x^3+x^2+x}dx = 2\ln|x| - 3\ln|x+1| + \arctan(x) + C$$

Which of the following is the partial fraction decomposition?

(a) $\frac{2}{x} - \frac{3x+1}{1+x^2}$ (b) $\frac{2}{x} - \frac{3}{x^2} + \frac{4}{1-x^2}$

(c) $\frac{2}{x} - \frac{3}{1+x} + \frac{1}{1+x^2}$ (d) $\frac{2}{x} + \frac{1-3x}{1+x^2}$

(e) $\frac{1}{x} + \frac{3}{1+x} + \frac{4}{1-x^2}$

16. ____________What is the interval of convergence for the series $\sum_{j=0}^{\infty}\left(\frac{1}{2}\right)^{j+1}\frac{x^{j+1}}{j+1}$?

(a) The series converges for all real numbers (b) $-2 < x < 2$

(c) The series converges only for $x = 0$. (d) $0 < x < 4$

(e) $-\frac{1}{2} < x < \frac{1}{2}$

17. ____________The partial fraction decomposition for $f(x) = \dfrac{x-1}{(x+1)^2}$ is

$$f(x) = \frac{1}{1+x} - \frac{2}{(1+x)^2}$$

Based on this decomposition, we know

(a) $\displaystyle\int f(x)dx = \ln|1+x| + \frac{2}{1+x} + C$

(b) $\displaystyle\int f(x)dx = \ln|1+x| - 2\ln(1+x)^2 + C$

(c) $\displaystyle\int f(x)dx = 2\ln|1+x|^3 + C$

(d) $\displaystyle\int f(x)dx = \arctan(1+x) + \frac{4}{(1+x)^3} + C$

(e) $\displaystyle\int f(x)dx = \frac{4}{(1+x)^3} - \arctan(x) + C$

18. ____________The trigonometric substitution required to evaluate $\displaystyle\int \frac{x^2}{\sqrt{64-9x^2}}dx$ would be

(a) $\tan\theta = \dfrac{3x}{8}$

(b) $\tan\theta = \dfrac{8x}{3}$

(c) $\sin\theta = \dfrac{3x}{8}$

(d) $\sin\theta = \dfrac{8x}{3}$

(e) $\tan\theta = x$

19. ____________After making the appropriate trigonometric substitution, it can be shown that

$$\int \frac{x^2}{\sqrt{64-9x^2}}dx = \frac{32}{27}\theta - \frac{32}{27}\sin(\theta)\cos(\theta) + C$$

Which of the following formulas correctly gives the antiderivative family as functions of $x$?

(a) $\displaystyle\int f(x)dx = \tfrac{32}{27}\arctan\left(\frac{3x}{8}\right) - \tfrac{1}{18}x\sqrt{64+9x^2} + C$

(b) $\displaystyle\int f(x)dx = \tfrac{32}{27}\arcsin\left(\frac{8x}{3}\right) - \tfrac{256}{81}x\sqrt{64-9x^2} + C$

(c) $\displaystyle\int f(x)dx = \tfrac{32}{27}\arctan\left(\frac{8x}{3}\right) - \tfrac{256}{81}x\sqrt{64+9x^2} + C$

(d) $\displaystyle\int f(x)dx = \tfrac{32}{27}\left[\arctan(x) - x\sqrt{64+9x^2}\right] + C$

(e) $\displaystyle\int \frac{x^2}{\sqrt{64-9x^2}}dx = \tfrac{32}{27}\arcsin\left(\frac{3x}{8}\right) - \tfrac{1}{18}x\sqrt{64-9x^2} + C$

20. ____________After making an appropriate substitution, what would the new limits be on the definite integral below?

$$\int_2^4 \frac{\sqrt{x}}{1+x^3}dx$$

(a) $u = 2\sqrt{2}$ and $u = 8$

(b) $u = 8$ and $u = 64$

(c) $u = \sqrt{2}$ and $u = 2$

(d) $u = 9$ and $u = 65$

(e) $u = 2$ and $u = 4$

21. ____________ What is the series representation for the function $f(x) = \dfrac{1}{4+x^2}$?

(a) $f(x) = \sum_{j=0}^{\infty}(-1)^j \left(\frac{1}{4}\right)^{j+1} x^{2j}$ (b) $f(x) = \sum_{j=0}^{\infty} (-1)^j x^{2j}$

(c) $f(x) = \frac{1}{4}\sum_{j=0}^{\infty} \left(\frac{1}{2}\right)^{j} x^{j}$ (d) $f(x) = \frac{1}{4}\sum_{j=0}^{\infty} x^{2j}$

(e) $f(x) = \sum_{j=0}^{\infty} (-1)^j \left(\frac{1}{4}\right)^{j} x^{j+2}$

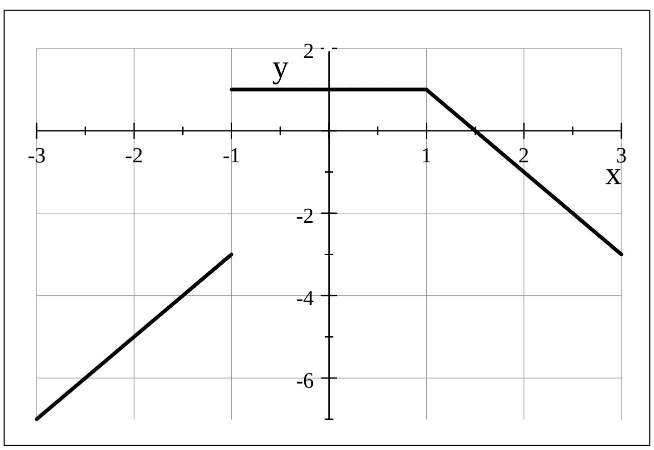
22. ____________ Since $\arctan\left(\frac{x}{2}\right) = 2\int_0^x \frac{1}{4+t^2}dt$, we know that a possible series representation for $\arctan\left(\frac{x}{2}\right)$ is

(a) $2\sum_{j=0}^{\infty} (-1)^j \frac{x^{j+3}}{j+3}$ (b) $\frac{1}{2}\sum_{j=0}^{\infty} \frac{x^{2j+1}}{2j+1}$

(c) $\sum_{j=0}^{\infty} \left(\frac{1}{2}\right)^{j+1} \frac{x^{j+1}}{j+1}$ (d) $2\sum_{j=0}^{\infty} (-1)^j \frac{x^{2j+1}}{2j+1}$

(e) $2\sum_{j=0}^{\infty}(-1)^j \left(\frac{1}{4}\right)^{j+1} \frac{x^{2j+1}}{2j+1}$

Consider the graph of the function $f$ shown below. Problems 23 and 24 refer to this graph.




23. ____________ What is the average value of the function $f$ on the interval $0 \le x \le 3$?

(a) $-3$ (b) $\frac{1}{3}$

(c) 3 (d) $-\frac{1}{3}$

(e) $-1$

24. ____________What is the value of the definite integral $\int_{2}^{-2} f(x)dx$?

(a) $-4$ (b) $4$

(c) $2$ (d) $-2$

(e) $-1$

25. ____________What can be said about the improper integral $\int_{-1}^{1} \frac{1}{(x+1)^{1/3}} dx$?

(a) The integral diverges. (b) The integral converges to $\frac{3}{\sqrt[3]{2}}$.

(c) The integral converges to $\frac{1}{\sqrt[3]{2}}$. (d) The integral converges to $\sqrt[3]{2}$.

(e) The integral converges to $\sqrt[3]{4}$.

**ANSWERS**

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|---|---|---|---|---|
| (1) E | (2) A | (3) D | (4) D | (5) C |
| (6) B | (7) A | (8) A | (9) E | (10) E |
| (11) E | (12) B | (13) C | (14) D | (15) C |
| (16) B | (17) A | (18) C | (19) E | (20) A |
| (21) A | (22) E | (23) D | (24) C | (25) B |