Math 3110 Practice Final

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Answers may be found at the end of this exam.

(1) ____________ What is the domain of $\mathbf{r} = \text{Arcsin}(t)\mathbf{i} + \ln(2t+1)\mathbf{j} + t\mathbf{k}$?

(a) $D = (-1/2, 1]$

(b) $D = [-1, 1/2)$

(c) $D = (-\infty, -1/2]$

(d) $D = [-1, 1]$

(e) $D = (-1/2, +\infty)$

(2) ____________ Suppose the acceleration of an object is given by $\mathbf{a} = -\mathbf{k}$. If we know $\mathbf{v}(1) = \mathbf{i} - \mathbf{j}$, then the velocity function for this object is given by

(a) $\mathbf{v} = (t-1)\mathbf{i} - t\mathbf{j} + (t^2 - 1)\mathbf{k}$

(b) $\mathbf{v}(t) = \mathbf{i} - t\mathbf{j}$

(c) $\mathbf{v}(t) = t\mathbf{i} - \mathbf{j}$

(d) $\mathbf{v}(t) = \mathbf{i} - \mathbf{j} + (1-t)\mathbf{k}$

(e) $\mathbf{v}(t) = 2(t-1)\mathbf{i} - 2t\mathbf{j} + (t^2 - 1)\mathbf{k}$

(3) ____________ Suppose that $\mathbf{v} = \langle 1, 3, 2 \rangle$. The component of $\mathbf{v}$ parallel to the vector $\mathbf{a} = \langle 1, 1, 1 \rangle$ is

(a) $\mathbf{u} = \langle -1, 1, 1 \rangle$

(b) $\mathbf{u} = \langle 2, 2, 2 \rangle$

(c) $\mathbf{u} = \langle 3, 6, 4 \rangle$

(d) $\mathbf{u} = \langle -2, -3, -2 \rangle$

(e) $\mathbf{u} = \dfrac{1}{\sqrt{14}} \langle 1, 3, 2 \rangle$

(4) ____________ Converting the integral below into polar coordinates gives us

$$\int_{x=0}^{1} \int_{y=\sqrt{1-x^2}}^{\sqrt{9-x^2}} dy\, dx + \int_{x=1}^{3} \int_{y=0}^{\sqrt{9-x^2}} dy\, dx$$

(a) $\displaystyle\int_{\theta=0}^{\pi/4} \int_{r=1}^{3} r\, dr\, d\theta$

(b) $\displaystyle\int_{\theta=\pi/4}^{\pi/2} \int_{r=1}^{3} r\, dr\, d\theta$

(c) $\displaystyle\int_{\theta=\pi/4}^{\pi/2} \int_{r=0}^{3} r\, dr\, d\theta$

(d) $\displaystyle\int_{\theta=0}^{\pi/4} \int_{r=0}^{3} r\, dr\, d\theta$

(e) $\displaystyle\int_{\theta=0}^{\pi/2} \int_{r=1}^{3} r\, dr\, d\theta$

(5) ____________ If a surface is defined by $f(x, y) = x^2 - y^2 + 6y$, then this surface will have a horizontal tangent plane at the point

(a) $P = (-1, 1, 6)$

(b) $P = (3, 0, 0)$

(c) $P = (0, -1, -6)$

(d) $P = (0, 0, 0)$

(e) $P = (0, 3, 9)$

(6) ____________ What is the region used in the integral $\displaystyle\int_{y=2}^{4} \int_{x=y-1}^{6} x^3 y\, dx\, dy$?

(a) a rectangle

(b) a trapezoid

(c) a square

(d) a triangle

(e) a circle

(7) ____________ Which of the following is the component of $\mathbf{v} = \langle 1, 1, 1\rangle$ that is perpendicular to $\mathbf{a} = \langle 2, 2, 2\rangle$?

(a) $\mathbf{u} = \mathbf{0}$

(b) $\mathbf{u} = \langle 1, 1, 1\rangle$

(c) $\mathbf{u} = \langle 2, 2, 2\rangle$

(d) $\mathbf{u} = \dfrac{1}{2\sqrt{3}}\langle 2, 2, 2\rangle$

(e) $\mathbf{u} = \dfrac{1}{\sqrt{2}}\langle 1, 0, -1\rangle$

(8) ____________ Suppose $\mathbf{v}$ and $\mathbf{a}$ are nonzero vectors that are not parallel. Which of the following is parallel to $\mathbf{v}$?

(a) $\mathbf{u} = \mathbf{v} \times \mathbf{a}$

(b) $\mathbf{u} = \mathbf{proj_a}(\mathbf{v})$

(c) $\mathbf{u} = \mathbf{proj_v}(\mathbf{a})$

(d) $\mathbf{u} = \mathbf{v} - \mathbf{proj_v}(\mathbf{a})$

(e) $\mathbf{u} = \mathbf{v} \cdot \mathbf{a}$

(9) ____________ Suppose that $\mathbf{u}$ and $\mathbf{v}$ are three-dimensional vectors. Which of the following vectors will be orthogonal to $\mathbf{u}$?

(a) $\mathbf{a} = \mathbf{u} - \mathbf{proj_u}(\mathbf{v})$

(b) $\mathbf{a} = \mathbf{u} \times \mathbf{v}$

(c) $\mathbf{a} = \mathbf{u} - \mathbf{proj_u}(\mathbf{v})$

(d) Both (a) and (b)

(e) Both (b) and (c)

(10) ____________ If $\mathbf{r} = (t^2 - 2t + 3)\mathbf{i} + (t-1)^3\mathbf{j}$, then $\mathbf{r}$ will fail to be smooth

(a) at no values of t

(b) at $t = 3$

(c) at $t = \pm 1$

(d) at $t = 0$

(e) at $t = 1$

(11) ____________ Suppose that $\mathbf{r} = \sqrt{t-1}\mathbf{i} - 2t^2\mathbf{j}$. Which of the following vectors are orthogonal to $\mathbf{r}(2)$?

(a) $\mathbf{v} = \langle 1, -1\rangle$

(b) $\mathbf{v} = \langle 8, 1\rangle$

(c) $\mathbf{v} = \langle 2, 3\rangle$

(d) $\mathbf{v} = \langle 0, 4\rangle$

(e) All of the above

(12) ____________ Suppose that the velocity of an object is given by $\mathbf{v}(t) = \sin(t)\mathbf{i} - 4t^3\mathbf{j}$. The acceleration for this object is

(a) $\mathbf{a}(t) = -\cos(t) - t^4\mathbf{j} + \mathbf{k}$

(b) $\mathbf{a}(t) = -\cos(t) - t^4\mathbf{j}$

(c) $\mathbf{a}(t) = \cos(t)\mathbf{i} - 12t^2\mathbf{j}$

(d) $\mathbf{a}(t) = -\sin(t)\mathbf{i} - t^5/5\mathbf{j} + \mathbf{k}$

(e) $\mathbf{a} = \dfrac{1}{\sqrt{16t^6 + \sin^2(t)}}\left(\sin(t)\mathbf{i} - 4t^3\mathbf{j}\right)$

(13) ____________ The equation of the plane passing through the points $P = (1, 4, 5)$, $Q = (3, -1, 2)$, and $R = (2, 1, 0)$ is

(a) $(x-1) - 3(y-4) - 5(z-5) = 0$

(b) $2(x-2) - 5(y-1) - 3z = 0$

(c) $16(x-3) + (y+1) + 9(z-2) = 0$

(d) $16(x-1) + 7(y-4) - (z-5) = 0$

(e) $(x-2) - 3(y+5) - 5(z+3) = 0$

(14) ______________ If $f(x, y, z) = x^2z - y^3$, then a vector pointing in the direction of maximum decrease at the point $(1, 2, 3)$ is

(a) $\mathbf{v} = \langle 3, -8, 3\rangle$

(b) $\mathbf{v} = \langle 6, -12, 1\rangle$

(c) $\mathbf{v} = \langle 6, -12, 1\rangle$

(d) $\mathbf{v} = \langle -6, 12, -1\rangle$

(e) $\mathbf{v} = \langle -2, 4, -3\rangle$

(15) ______________ If $\mathbf{r} = sin(\pi t)\mathbf{i} + t^2\mathbf{j} - \cos(\pi t)\mathbf{k}$, then the unit tangent vector to $\mathbf{r}$ at $t = 1$ is given by

(a) $\mathbf{T} = \dfrac{\mathbf{j} - \pi\mathbf{k}}{\pi}$

(b) $\mathbf{T} = \dfrac{2\mathbf{j} + \pi\mathbf{k}}{\sqrt{\pi^2 + 4}}$

(c) $\mathbf{T} = \dfrac{\mathbf{i} + \mathbf{j}}{\pi}$

(d) $\mathbf{T} = \dfrac{-\mathbf{i} + 2\mathbf{j}}{2 + \pi}$

(e) $\mathbf{T} = \dfrac{-\pi\mathbf{i} + 2\mathbf{j}}{\sqrt{\pi^2 + 4}}$

(16) ______________ Which of the following curves DOES NOT represent a circle?

(a) $\mathbf{r}(t) = \cos(3t)\mathbf{i} + \sin(3t)\mathbf{j}$

(b) $\mathbf{r}(t) = 3\cos(t)\mathbf{i} - 4\sin(t)\mathbf{j}$

(c) $\mathbf{r}(t) = \sqrt{4 - t^2}\mathbf{i} + t\mathbf{j}$

(d) Neither (a) nor (b)

(e) Neither (b) nor (c)

(17) ______________ If $f(x, y, z) = \sin(xy)\cos(yz)$, then

(a) $f_z(x, y, z) = -y\sin(xy)\sin(yz)$

(b) $f_z(x, y, z) = x\cos(xy)\sin(yz) - y\sin(xy)\sin(yz)$

(c) $f_z(x, y, z) = y\cos(xy)\sin(yz)$

(d) $f_z(x, y, z) = -xy\cos(xy)\cos(yz)$

(e) $f_z(x, y, z) = y^2\cos(xy)\cos(yz)$

(18) ______________ If $f(x, y) = x^2 - 2xy + y^2$, then

(a) $f_{yx}(x, y) = 2x$

(b) $f_{yx}(x, y) = y$

(c) $f_{yx}(x, y) = -2$

(d) $f_{yx}(x, y) = 0$

(e) $f_{yx}(x, y) = 2y$

(19) ______________ If $f(x, y) = y\arctan(x)$, then the directional derivative for f at $(1, 1)$ in the direction of $\mathbf{v} = \mathbf{i} - \mathbf{j}$ is given by

(a) $D_{\mathbf{v}}(1, 1) = \dfrac{1}{2}\left(\dfrac{1}{\sqrt{2}} + \dfrac{\pi}{2}\right)$

(b) $D_{\mathbf{v}}(1, 1) = 0$

(c) $D_{\mathbf{v}}(1, 1) = \dfrac{1}{4}$

(d) $D_{\mathbf{v}}(1, 1) = \dfrac{1}{2}\left(\dfrac{1}{2} + \dfrac{\pi}{3}\right)$

(e) $D_{\mathbf{v}}(1, 1) = \dfrac{1}{\sqrt{2}}\left(\dfrac{1}{2} + \dfrac{\pi}{4}\right)$

(20) _____________ If $f(x,y) = x^2 - 2xy + y^2$, then the differential for f is given by

(a) $dz = 2(x-y)dx + 2(y-x)dy$

(b) $dz = 2ydx - 2xdy$

(c) $dz = (2x-y)dx + (2y-x)dy$

(d) $dz = (2y-x)dx + (2x-y)dx$

(e) $dz = y^2dx + x^2dy$

(21) _____________ The volume of a cylinder is given by $V = \pi r^2 h$. If measurement of a particular cylinder show $r = 4 \pm .01$ inches and $h = 7 \pm .02$ inches, then the maximum error in the volume resulting from the measurements is

(a) Approximately 1.28π

(b) Approximately $.88\pi$

(c) Approximately π

(d) Approximately $.4\pi$

(e) Approximately $.32\pi$

(22) _____________ The parametric equations for the line passing through the points $P = (1,4,5)$ and $Q = (3,-1,2)$ will be

(a) $x = 2+t \quad y = 5-4t \quad z = -3-5t$

(b) $x = 2-3t \quad y = -5+t \quad z = -3-2t$

(c) $x = 1+2t \quad y = 4-5t \quad z = 5-3t$

(d) $x = t \quad y = 4t \quad z = 5t$

(e) $x = -3t \quad y = t \quad z = -2t$

(23) _____________ Suppose f is a function of two variables with continuous second partial derivatives, and let

$$D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - [f_{xy}(x,y)]^2$$

The function f will have a local maximum at a point $P = (a,b)$ if

(a) $D(a,b) > 0$ and $f_{xx}(a,b) < 0$

(b) $D(a,b) < 0$ and $f_{xx}(a,b) < 0$

(c) $D(a,b) > 0$ and $f_{xx}(a,b) > 0$

(d) $D(a,b) < 0$ and $f_{xx}(a,b) > 0$

(e) $D(a,b) > 0$ and $f_{xy}(a,b) > 0$

(24) _____________ Subject to the constraint $x + 2y - 5 = 0$, the minimum value of $f(x,y) = x^2 + y^2$ occurs at the point

(a) $P = (4, 1/2)$

(b) $P = (-1,3)$

(c) $P = (1,2)$

(d) $P = (5,0)$

(e) $P = (-3,4)$

(25) _____________ Suppose we want to use the transformation $x = v\sqrt{u}$ and $y = u\sqrt{v}$ on a particular integral. The absolute value of the Jacobian for this transformation will be

(a) $\left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| = uv$

(b) $\left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| = \sqrt{uv}$

(c) $\left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| = \dfrac{3uv}{2}$

(d) $\left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| = \dfrac{3\sqrt{uv}}{4}$

(e) $\left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| = \dfrac{2}{3}$

(26) ______________ The function $z = x^2 - y^2 - xy - 3x - y$ has a saddle point at

(a) $P = (-1, 1)$

(b) $P = (1, -1)$

(c) $P = (-1, -1)$

(d) $P = (2, 3)$

(e) $P = (-2, 3)$

(27) ______________ Switching the order of integration on the integral below gives us

$$\int_{x=1}^{3} \int_{y=x}^{x^2} dy\, dx$$

(a) $\displaystyle\int_{y=1}^{9} \int_{x=\sqrt{y}}^{3} dx\, dy$

(b) $\displaystyle\int_{y=1}^{9} \int_{x=3}^{\sqrt{y}} dx\, dy$

(c) $\displaystyle\int_{y=1}^{9} \int_{x=\sqrt{y}}^{y} dx\, dy$

(d) $\displaystyle\int_{y=1}^{\sqrt{3}} \int_{x=\sqrt{y}}^{y} dx\, dy + \int_{y=\sqrt{3}}^{9} \int_{x=\sqrt{y}}^{3} dx\, dy$

(e) $\displaystyle\int_{y=1}^{2} \int_{x=y}^{2} dx\, dy + \int_{y=2}^{9} \int_{x=\sqrt{y}}^{3} dx\, dy$

(28) ______________ Evaluating $\displaystyle\int_{y=0}^{4\pi} \int_{x=1}^{\sin(y)} dx\, dy$ gives us

(a) -4π

(b) $2 - 4\pi$

(c) $-(2 + 4\pi)$

(d) $4\pi - 2$

(e) -2

(29) ______________ The integral below is presented in cylindrical coordinates. How would it appear in rectangular coordinates?

$$\int_{r=0}^{4} \int_{\theta=0}^{\pi/4} \int_{z=0}^{r} r^3 \cos^2\theta\, dz\, d\theta\, dr$$

(a) $\displaystyle\int_{x=0}^{2\sqrt{2}} \int_{y=0}^{\sqrt{16-y^2}} \int_{z=\sqrt{x^2+y^2}}^{x^2+y^2} x^2\sqrt{x^2+y^2}\, dz\, dy\, dx$

(b) $\displaystyle\int_{x=0}^{2\sqrt{2}} \int_{y=x}^{\sqrt{16-y^2}} \int_{z=0}^{\sqrt{x^2+y^2}} y^2\, dz\, dy\, dx$

(c) $\displaystyle\int_{y=0}^{2\sqrt{2}} \int_{x=y}^{\sqrt{16-y^2}} \int_{z=0}^{\sqrt{x^2+y^2}} x^2\, dz\, dx\, dy$

(d) $\displaystyle\int_{y=0}^{2\sqrt{2}} \int_{x=0}^{y} \int_{z=\sqrt{16-y^2}}^{\sqrt{x^2+y^2}} xz\sqrt{x^2+y^2}\, dz\, dx\, dy$

(e) $\displaystyle\int_{y=0}^{2\sqrt{2}} \int_{x=0}^{y} \int_{z=0}^{\sqrt{x^2+y^2}} x^2\, dz\, dx\, dy + \int_{y=2\sqrt{2}}^{4} \int_{x=0}^{\sqrt{16-y^2}} \int_{z=0}^{\sqrt{x^2+y^2}} x^2\, dz\, dx\, dy$

(30) ______________ If $f(x, y) = x\sin(y)$, then $f_u(uv, v - u)$ is equal to

(a) $-u\sin(v - u)$

(b) $v\cos(v - u) + u\sin(v - u)$

(c) $uv\sin(v - u)$

(d) $-v\cos(v - u)$

(e) $v(sin(v - u) - u\cos(v - u)$

ANSWERS

(1) A	(7) A	(13) D	(19) E	(25) D
(2) D	(8) C	(14) D	(20) A	(26) B
(3) B	(9) D	(15) E	(21) B	(27) D
(4) E	(10) E	(16) B	(22) C	(28) A
(5) E	(11) B	(17) A	(23) A	(29) C
(6) B	(12) C	(18) C	(24) C	(30) E