REVIEW FOR EXAM II

The video on Page 37 of your online text will be helpful on this review, especially for Problem 6. You will need to write your answers on a separate sheet of paper. There is not room on this sheet for your answers.

1. Suppose that the function h is defined by the output formula $u = h(m) = \dfrac{4m+1}{5}$.

   (a) What is the domain for the function h?

   Solution. The domain will be the set of all real numbers, because the formula contains no square roots or division by variable expressions.

   (b) Construct the output formula for the function h^{-1}. Use proper function notation in your answer.

   Solution. The simplest way to determine the output formula for the inverse function is to solve the given output formula for the variable m. Observe

$$u = \frac{4m+1}{5} \Longrightarrow 5u = 4m+1 \Longrightarrow 5u-1 = 4m \Longrightarrow \frac{5u-1}{4} = m$$

   This relation defines the output formula for the inverse function. In particular,

$$m = h^{-1}(u) = \frac{5u-1}{4}$$

   (c) Suppose the function g is defined by the output formula $b = g(u) = \dfrac{1}{u}$. Construct the output formulas for $g \circ h$ and for $h^{-1} \circ g$. Simplify your formulas as much as possible.

   Solution. The output formula for $g \circ h$ will be $b = g(h(m))$. Now, observe

$$b = g(h(m)) \Longrightarrow b = g\left(\frac{4m+1}{5}\right) \Longrightarrow b = \frac{1}{\frac{4m+1}{5}} \Longrightarrow b = \frac{5}{4m+1}$$

   The output formula is therefore $b = g(h(m)) = \dfrac{5}{4m+1}$.
   The output formula for $h^{-1} \circ g$ will be $m = h^{-1}(g(u))$. Now, observe

$$\begin{aligned}
m = h^{-1}(g(u)) \quad &\Longrightarrow \quad m = h^{-1}\left(\frac{1}{u}\right) \\
&\Longrightarrow \quad m = \frac{5(1/u)-1}{4} \\
&\Longrightarrow \quad m = \frac{5/u-1}{4} \\
&\Longrightarrow \quad m = \frac{\frac{5-u}{u}}{4} \\
&\Longrightarrow \quad m = \frac{5-u}{4u}
\end{aligned}$$

   The output formula is therefore $m = h^{-1}(g(u)) = \dfrac{5-u}{4u}$.

2. Suppose the function g is defined by the output formula $y = g(x) = \dfrac{2x}{4+x}$.

(a) If f is defined by the output formula $t = f(w) = \sqrt{w}$, then construct the output formulas for $f \circ g$ and $g \circ f$. Simplify your formulas as much as possible.

Solution. The output formula for $f \circ g$ is $t = f(g(x))$. Observe

$$t = f(g(x)) \Longrightarrow t = f\left(\frac{2x}{4+x}\right) \Longrightarrow t = \sqrt{\frac{2x}{4+x}}$$

Therefore, the output formula is $t = f(g(x)) = \sqrt{\dfrac{2x}{4+x}}$.

Solution. The output formula for $g \circ f$ is $y = g(f(w))$. Observe

$$y = g(f(w)) \Longrightarrow y = g\left(\sqrt{w}\right) \Longrightarrow y = \frac{2\sqrt{w}}{4+\sqrt{w}}$$

Therefore, the output formula for $g \circ f$ is $y = g(f(w)) = \dfrac{2\sqrt{w}}{4+\sqrt{w}}$.

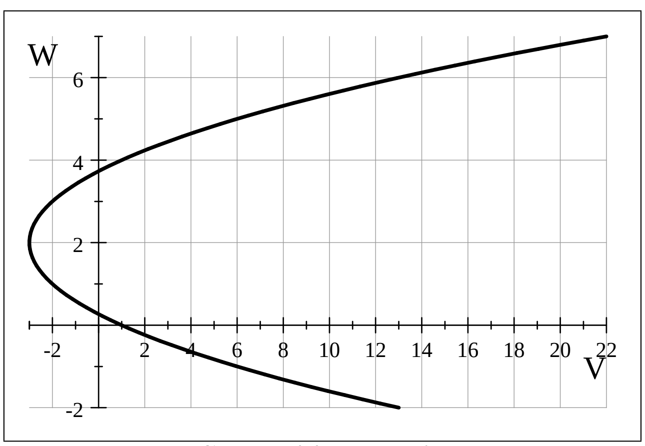
(b) Construct the output formula for the function g^{-1}.

Solution. Observe that

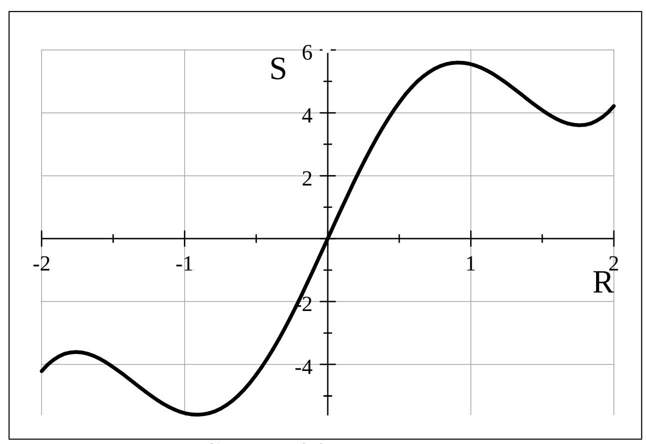
$$\begin{aligned} y = \frac{2x}{4+x} \quad &\Longrightarrow \quad y(4+x) = 2x \\ &\Longrightarrow \quad 4y + yx = 2x \\ &\Longrightarrow \quad 4y = 2x - yx \\ &\Longrightarrow \quad 4y = x(2-y) \\ &\Longrightarrow \quad \frac{4y}{2-y} = x \end{aligned}$$

Therefore, the inverse function is defined by the output formula $x = g^{-1}(y) = \dfrac{4y}{2-y}$.

3. The graphs below define *functions* f and g.

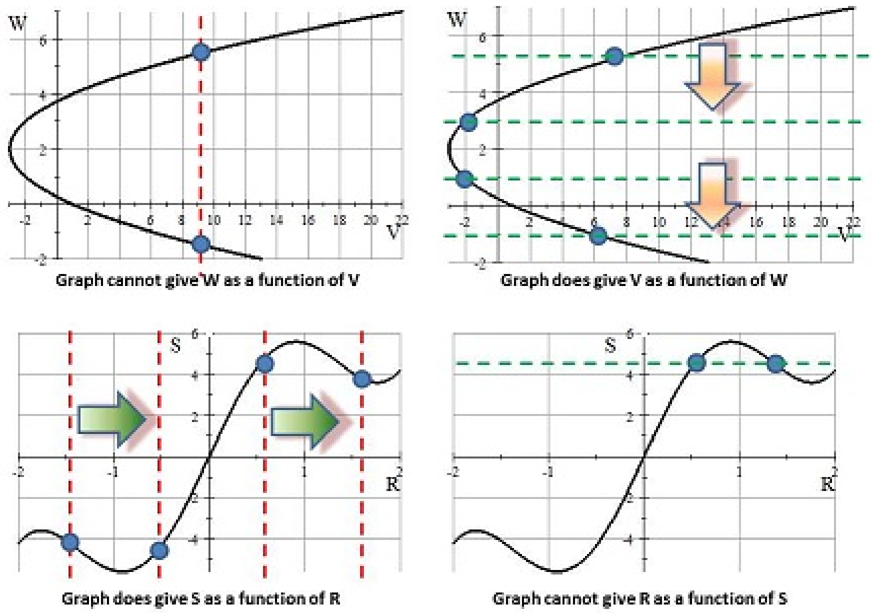


Graph of function f



Graph of function g

(a) What is the input variable for the function f? How do you know?



Solution. The graph on the left passes the horizontal line test but not the vertical line test. Since every input of a *function* can be paired with only one output, we have to conclude that W is the input variable.

(b) What is the input variable for the function g? How do you know?

Solution. The graph on the right passes the vertical line test but not the horizontal line test. Since every input of a *function* can be paired with only one output, we have to conclude that R is the input variable.

(c) If possible, use the graph to evaluate $f(f(3))$. If it is not possible, explain why.

Solution. We must read the input $W = 3$ off the *vertical* axis. According to the graph, $f(3) = -2$, and $f(-2) = 13$. Therefore, $f(f(3)) = 13$.

(d) If possible, use the graph to evaluate $f(g(2))$.

Solution. We must read the input $R = 2$ off the *horizontal* axis. According to the graph, $g(2) \approx 4.1$. Now, we must read $W = 4.1$ off the *vertical* axis on the graph of f. We see that $f(4.1) \approx 2$.

One could argue that the composition $f(g(2))$ has no meaning, since the output variable for g is called S, while the input variable for f is called W. This could be a valid argument if these variables had a specific meaning assigned to them (for example, one could represent temperature in degrees Fahrenheit while the other represents volume in cubic inches). However, there is no context for either variable, and hence *we can consider them to be interchangeable.*

(e) Use the graph to find all solutions to the equation $4 = g(x)$.

Solution. Since 4 is an output from the function g, we begin by drawing a horizontal line through $S = 4$ on the output axis for g. This line intersects the graph of g twice, namely at the points $(0.5, 4)$ and $(1.5, 4)$. Therefore, there are two solutions to this equation, namely $R = 0.5$ and $R = 1.5$.

4. The tables below provide information about the output for two functions f and g.

Input Value	Output from f
2	5
4	4
5	8
10	6

Input Value	Output from g
0	10
5	6
10	3
8	0

(a) What is the value of $f(f(2))$? Based on the table, we know $f(2) = 5$ and $f(5) = 8$. Therefore, $f(f(2)) = f(5) = 8$.

(b) What is the value of $g^{-1}(f(10))$? Based on the table, we know $f(10) = 6$ and $g(5) = 6$. Therefore, $g^{-1}(f(10)) = g^{-1}(6) = 5$.

(c) What is the value of $f^{-1}(g^{-1}(6))$? Based on the table, we know $g(5) = 6$, so $g^{-1}(6) = 5$. Likewise, since $f(2) = 5$, we know that $f^{-1}(5) = 2$. Therefore, $f^{-1}(g^{-1}(6)) = f^{-1}(5) = 2$.

5. Let g be any function that gives the values of y in terms of the values of x.

(a) Use proper function notation to write the formula that gives the average rate of change for y as x goes from $x = -3$ to $x = 1$.

Solution. The average rate of change will be given by

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-3)}{1 - (-3)} = \frac{g(1) - g(-3)}{4}$$

(b) Suppose now that g is the function in Problem 2. Compute the average rate of change for y as x goes from $x = -3$ to $x = 1$.

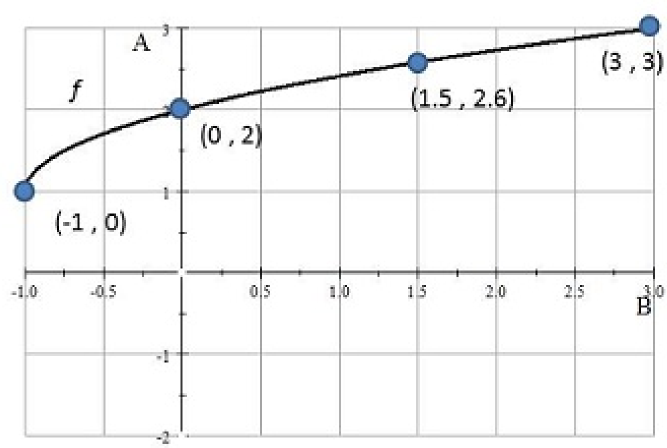
Solution. The function we are working with is defined by the output formula $y = g(x) = \dfrac{2x}{4+x}$. Now,

$$g(1) = \frac{2(1)}{4+1} = \frac{2}{5} \qquad g(-3) = \frac{2(-3)}{4-3} = -6$$

Therefore, the average rate of change will be

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-3)}{4} = \frac{2/5 + 6}{4} = \frac{32/5}{4} = \frac{32}{20} = \frac{8}{5}$$

6. The diagram below shows the graph of a function f that gives the values of A in terms of the values of B. On the grid provided, sketch the graph of f^{-1}. Be sure to label your axes.



Solution. The inverse function for the function f will take the output variable for f as its input variable. In essence, this means the graph of the inverse function can be obtained from the graph of f simply by swapping the coordinates of points on the graph of f. Here is a sketch of the graph of f^{-1}. Note the relabling of the axes.