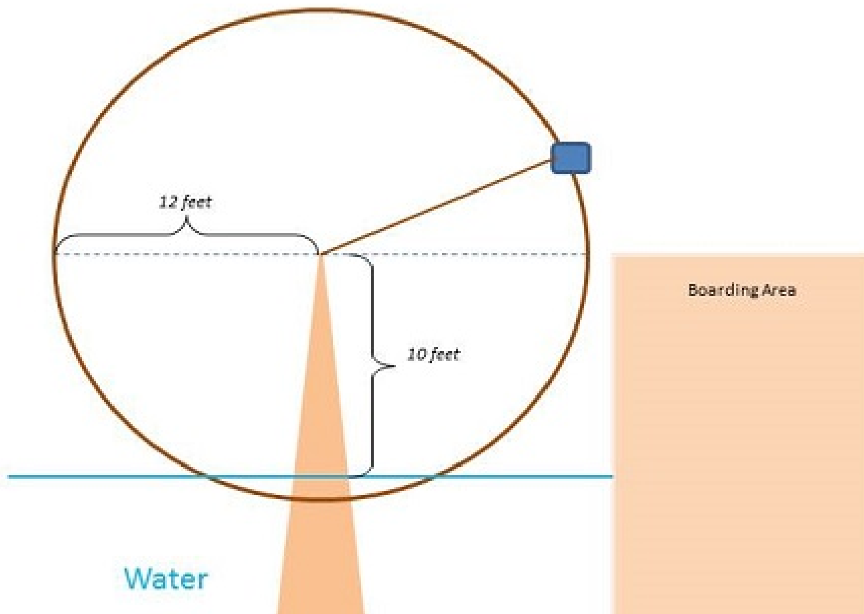
PRACTICE QUESTIONS FOR EXAM IV

An amusement park has a water ride set up in the form of a dunking wheel as shown in the diagram below. People get into a gondola at the boarding station and rotate counterclockwise around the wheel. Three full rotations makes up one ride on the wheel.



1. If it takes two minutes for the gondola to complete two-thirds of one complete rotation, what is the angular speed of the water wheel?

   Solution. Once the gondola completes two-thirds of one complete rotation, the angle determined by its initial and terminal positions will have radian measure $\frac{4\pi}{3}$ $\texttt{rad}$. Thus, the angular speed for the water wheel will be

$$\frac{\frac{4\pi}{3}\ \texttt{rad}}{2\ \texttt{minutes}} = \frac{2\pi}{3}\ \frac{\texttt{rad}}{\texttt{min}}$$

2. How far will the gondola travel around the water wheel in one complete rotation?

   Solution. Since the radius of the water wheel is twelve feet long, the gondola will travel $(12\ \texttt{feet})(2\pi) = 24\pi\ \texttt{feet}$.

3. How far will the gondola travel around the water wheel during seven-eighths of one ride?

   Solution. Over one complete ride, the gondola will travel $72\pi\ \texttt{feet}$. Thus, the distance it will travel over seven-eighths of one ride will be

$$\left(\frac{7}{8}\right)(72\pi\ \texttt{feet}) = 63\pi\ \texttt{feet}$$

4. Suppose the gondola has traveled four-fifths of its first complete rotation.

   (a) What is the radian measure of the angle it has rotated through?

   Solution. The radian measure of the angle determined by the gondola's initial and terminal position will be

$$\left(\frac{4}{5}\right)(2\pi\ \texttt{rad}) = \frac{8\pi}{5}\ \texttt{rad}$$

(b) What is the slope of the beam connecting the gondola to the center of the water wheel?

Solution. The slope of the beam is $\tan\left(\frac{8\pi}{5}\right) \approx -3.078$.

5. Suppose the ride is three-fifths over.

(a) What is the radian measure of the angle the gondola has rotated through?

Solution. One ride produces an angle of 6π `rad` between the initial and terminal positions (which coincide). Consequently, the gondola will move through an angle of

$$\left(\frac{3}{5}\right)(6\pi\ \texttt{rad}) = \frac{18\pi}{5}\ \texttt{rad}$$

(b) What are the x and y coordinates of the gondola?

Solution. The coordinates of the gondola will be

$$x = (12\ \texttt{feet})\cos\left(\frac{18\pi}{5}\right) \approx 3.71\ \texttt{feet} \qquad y = (12\ \texttt{feet})\sin\left(\frac{18\pi}{5}\right) = -11.41\ \texttt{feet}$$

(c) How high above (or below) the water is the gondola?

Solution. Since the y-coordinate for the gondola is 12 `feet`, the gondola will be

$$-11.41\texttt{ft} + 10\texttt{ft} = -1.41\ \texttt{feet}$$

above the water. (In other words, the gondola will be 1.41 feet *below* the water.)

(d) How far to the left of its starting point is the gondola?

Solution. Since the x-coordinate for the gondola is 3.71 `feet`, the gondola will be

$$12\texttt{ft} - 3.71\texttt{ft} = 8.29\ \texttt{feet}$$

to the left of its starting point.

6. Consider the position where the gondola first touches the water.

(a) What is the x coordinate of the gondola at this point?

Solution. When the gondola first touches the water, we know that it will be ten feet below the x-axis. This tells us that its y-coordinate will be $y = -10$ `feet`. Since the length of the beam connecting the gondola to the center of the water wheel is twelve feet long, and since the gondola will be in Quadrant III when it first touches the water, we know that

$$x = -\sqrt{(12\ \texttt{ft})^2 - (-10\ \texttt{ft})^2} = -2\sqrt{11}\ \texttt{ft}$$

(b) How far to the left of its initial point will the gondola be at this point?

Solution. The gondola will be

$$12\texttt{ft} - (-2\sqrt{11}\texttt{ft}) \approx 18.63\ \texttt{feet}$$

left of its initial point.

(c) Let θ be the radian measure of the angle formed by the initial and terminal rays at this point. What is the value of $\tan(\theta)$?

Solution. We know that

$$\tan(\theta) = \frac{-10 \texttt{ ft}}{-2\sqrt{11} \texttt{ ft}} = \frac{5}{\sqrt{11}}$$

7. When the gondola is at point $P = (x, y)$ on the water wheel, the beam connecting the gondola to the center has slope $m = 3.14$. If we also know that the gondola is *rising* as it passes this point, what are the coordinates for P?

Solution. Let θ be a radian measure of the angle determined by the initial and terminal positions of the gondola. Since the tangent of θ is positive, we know that the angle is either in Quadrant I or Quadrant III; and since the gondola is *rising*, we must conclude that the angle lies in Quadrant I. (With counter-clockwise rotation, the gondola would be *falling* in Quadrant III.) Consequently, if Q is any point on the terminal ray (the beam) of this angle, we know that both coordinates of Q must be positive. With this in mind, we know

$$\tan(\theta) = \frac{3.14}{1}$$

so the point $Q = (1, 3.14)$ must lie on the beam. Now, this point is a distance

$$r = \sqrt{(1 \texttt{ ft})^2 + (3.14 \texttt{ ft})^2} \approx 3.296 \texttt{ ft}$$

from the center of the water wheel. Unfortunately, this means the point Q does not correspond to the gondola, since the gondola must be twelve feet from the center. However, we do know that

$$\cos(\theta) \approx \frac{1 \texttt{ ft}}{3.296 \texttt{ ft}} \qquad \sin(\theta) \approx \frac{3.14 \texttt{ ft}}{3.296 \texttt{ ft}}$$

We are now able to determine the coordinates of the point P that corresponds to the gondola. Observe

$$x = (12 \texttt{ ft}) \cdot \left(\frac{1 \texttt{ ft}}{3.296 \texttt{ ft}} \right) \approx 3.64 \texttt{ feet}$$

$$y = (12 \texttt{ ft}) \cdot \left(\frac{3.14 \texttt{ ft}}{3.296 \texttt{ ft}} \right) \approx 11.43 \texttt{ feet}$$

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