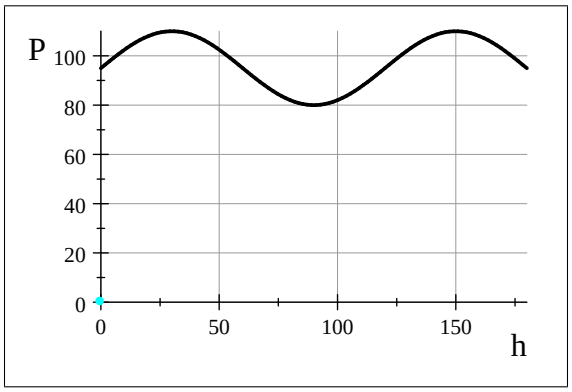
MATH 1730 EXAM II

(Version 1)

100 points

NAME:_____

Please place the letter of your selection in the blank provided. These questions are worth five points each. Problems 1 and 2 refer to the graph below. The graph gives Jerome's weight in pounds as a function f of the number n days after January 1, 2012.



5 pts 1. **B** Which of the following statements is a correct interpretation of $117 = f(140)$?

(a) The expression means 117 equals f times 140.

(b) The expression means an input of 140 days since January 1, 2012 produces an output of 117 pounds.

(c) The expression means an input of 140 pounds produces an output of 117 days since January 1, 2012.

(d) Both Statements (a) and (b) are true.

(e) Both Statements (a) and (c) are true.

5 pts 2. **C** What is the solution to the equation $80 = f(n)$?

(a) $n = \dfrac{90}{f}$ days since January 1, 2012 (b) $n \approx 82$ pounds

(c) $n \approx 90$ days since January 1, 2012 (d) Both Statements (a) and (b) are true.

(e) Both Statements (a) and (c) are true.

5 pts 3. **E** If $f(t) = \dfrac{2}{t}$ and $g(u) = u - 1$, then what is the rule defining the function $h = g \circ f$?

(a) $h(u) = \left(\dfrac{2}{u}\right)(u-1)$ (b) $h(t) = \left(\dfrac{2}{t}\right)(t-1)$

(c) $h(u) = \dfrac{2}{u-1}$ (d) $h(u) = \dfrac{2}{u-1} - 1$

(e) $h(t) = \dfrac{2}{t} - 1$

5 pts 4. **B** After completing a pit stop, a racecar pulls back onto the racetrack. Let f be the function that gives the distance in feet that race car has traveled since leaving the pit with respect to the time t in seconds since the racecar left the pit. Which of the following statements is true about the expression $f(5) - f(3)$?

I. It is true that $f(5) - f(3) = f(2)$.

II. It is true that $f(5) - f(3)$ is the change in distance traveled from $t = 3$ to $t = 5$ seconds since leaving the pit.

III. It is true that $f(5) - f(3)$ is the change in distance traveled whenever $\triangle t = 2$ seconds.

(a) Only Statement I is true.
(b) Only Statement II is true.
(c) Statements I and II are true.
(d) Statements II and III are true.
(e) All three statements are true.

5 pts 5. **C** Suppose that $y = f(x)$ defines the values of y as a function of the values of x. Which of the following expressions gives the average rate of change for f as the values of x change from $x = -2$ to $x = 5$?

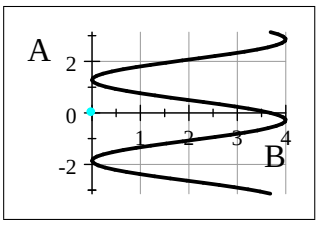
(a) $\dfrac{f(5) + f(2)}{3}$
(b) $f(5) + f(2)$
(c) $\dfrac{f(5) - f(-2)}{7}$
(d) 7
(e) $\dfrac{f(-2) - f(5)}{3}$

5 pts 6. **A** For the table below, which of the following statements is correct?

x	0.0	1.0	7.5	1.0
y	1.0	2.0	2.0	7.5

(a) In the relation, x is **not** a function of y, and y is **not** a function of x.
(b) In the relation, x is a function of y, and y is a function of x.
(c) In the relation, x is a function of y, but y is **not** a function of x.
(d) In the relation, x is **not** a function of y, but y is a function of x.
(e) There is not enough information to tell whether one variable is a function of the other.

5 pts 7. **B** Which of the following statements is true about the relationship shown in the graph below?



(a) A is a function of B.
(b) B is a function of A.
(c) Both (a) and (b) are true.
(d) Neither (a) nor (b) are true.
(e) There is not enough information to determine.

5 pts 8. **C** If $y = h(x) = 3x + 1$, then what steps must be followed to create the rule defining the function h^{-1}?

(a) Take the reciprocal of y.

(b) Divide y by 3, then subtract 1 from the result.

(c) Subtract 1 from y, then divide the result by 3.

(d) Add 3 to y, then multiply the result by x.

(e) Take the reciprocal of y, multiply the result by 3, and then subtract 1.

Problems 9, 10, and 11 refer to the tables below.

Value of x	Value of $f(x)$
1	4.0
4	5.0
6	10.2
7	9.7
8	11.3

Table for the function f

Value of u	Value of $g(u)$
1.0	4.0
1.5	9.1
2.5	9.1
4.0	9.7
5.0	10.2

Table for the function g

5 pts 9. **D** Based on the tables, what is the value of $f^{-1}(g(5.0))$?

(a) 10.2

(b) 4

(c) $\frac{1}{4}$

(d) 6

(e) $\frac{1}{10.2}$

5 pts 10. **E** Based on the tables, what is the value of $g(g(1.0))$?

(a) 10.2

(b) 104.04

(c) 5.0

(d) 6

(e) 9.7

5 pts 11. **D** Based on the tables, what is true about the expression $g(f(4))$?

(a) $g(f(4)) = 5.0$

(b) $g(f(4)) = 9.7$

(c) $g(f(4)) = 4.0$

(d) $g(f(4)) = 10.2$

(e) $g(f(4))$ cannot be computed using this table.

10 pts 12. If $w = h(t) = \dfrac{5t+2}{7}$, construct the inverse function for h. You must show your steps and use proper function notation for full credit.

Solution. There are two ways to approach this problem ...both of which are really equivalent. We could list the steps in the process that transforms the values of t into the values of w and then reverse those steps to determine the output formula for h^{-1}. We could also solve the equation relating w and t for the variable t.

THE STEP METHOD

The process named h is

STEP 1: Take the value of t and multiply it by 5.

STEP 2: Take the result of Step 1 and add 2 to it.

STEP 3: Take the result of Step 2 and divide it by 7.

The process named h^{-1} is

STEP 1: Take the value of w and multiply it by 7.

STEP 2: Take the result of Step 1 and subtract 2 from it.

STEP 3: Take the result of Step 2 and divide it by 5.

Translating these steps into algebra gives us the output formula for h^{-1}. In particular, $t = h^{-1}(w) = \dfrac{7w-2}{5}$.

SOLVING THE EQUATION FOR t

$$\begin{aligned} w = \frac{5t+2}{7} &\implies 7w = 5t+2 \\ &\implies 7w-2 = 5t \\ &\implies \frac{7w-2}{5} = t \end{aligned}$$

Once again, we see that the output formula for $h^{-1}$is given by $t = h^{-1}(w) = \dfrac{7w-2}{5}$.

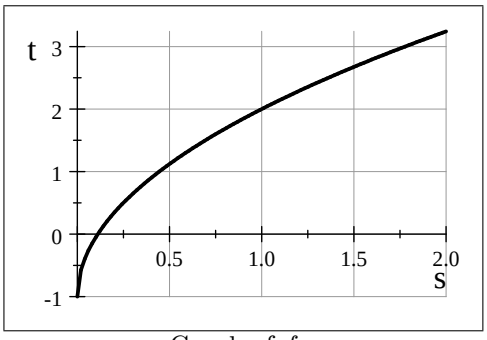
10 pts 13. Construct the output formula for the function $g \circ f$ if we know that $y = f(x) = 2\sqrt{x}$ and $u = g(y) = \dfrac{y-1}{y}$. You must use proper function notation for full credit.

Solution. Based on the output formulas given, we know

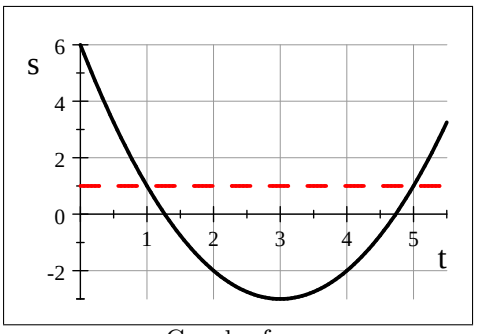
$$u = g(f(x)) \implies u = g\left(2\sqrt{x}\right) \implies u = \frac{2\sqrt{x}-1}{2\sqrt{x}}$$

Therefore, the output formula for $g \circ f$ is $u = g(f(x)) = \dfrac{2\sqrt{x}-1}{2\sqrt{x}}$.

14. The function f below gives t as a function of s, and the function g below gives s as a function of t.



Graph of f



Graph of g

5 pts (a) If possible, evaluate $g(f^{-1}(1))$. If it is not possible, explain why.

Solution. Based on the graphs given, we see that the function f has an inverse ... because its graph passes the horizontal line test. Now, $f(0.5) = 1$, so we know that $f^{-1}(1) = 0.5$. Therefore, $g(f^{-1}(1)) = g(0.5) = 4$.

5 pts (b) If possible, evaluate $f(g^{-1}(1))$. If it is not possible, explain why.

Solution. The graph of the function g fails the horizontal line test; we must therefore conclude that g does not have an inverse. It is not possible to evaluate this expression.

5 pts (c) If possible, find all solutions to the equation $f(g(t)) = 2$ for the variable t. If it is not possible, explain why.

Solution. The equation $f(g(t)) = 2$ tells us we must have $g(t) = 1$, since $f(1) = 2$. The dashed line $s = 1$ shown above intersects the graph of g at two points, namely $(1, 1)$ and $(5, 1)$. Consequently, there are two solutions to the equation, namely $t = 1$ and $t = 5$.

10 pts 15. What is the domain of the function f defined by the output formula $y = f(x) = \dfrac{\sqrt{1-3x}}{x+2}$? You must show your work for full credit.

Solution. First, notice that there will be division by 0 when $x + 2 = 0$, and this occurs when $x = -2$. We therefore know that $x = -2$ must be excluded from the domain of the function. Next, observe that there will be square roots of negative numbers when $1 - 3x < 0$, and this occurs when $x > 1/3$. We must therefore exclude all values of x that are larger than $1/3$ from the domain of the function. The domain of f will therefore be all $x \leq 1/3$ EXCEPT for $x = -2$.