MATH 1730 EXAM II
(Retake)
100 points

NAME:_____

10 pts 1. Suppose that two functions f and g are defined by output formulas $y = f(x) = 2x+2$ and $s = g(t) = 4t^2$. Construct the output formula for the function $g \circ f$. You must use proper function notation, expand and simplify your answer for full credit.

Solution. The output formula will be

$$
\begin{aligned}
s = (g \circ f)(x) &= g(f(x)) \\
&= g(2x + 2) \\
&= 4(2x + 2)^2 \\
&= 4(4x^2 + 8x + 4) \\
&= 16x^2 + 32x + 16
\end{aligned}
$$

2. Suppose that k is a function that gives m in terms of n

5 pts (a) What is the input variable for the function k? *The input variable must be n.*

5 pts (b) When the input into k is 3, the output from k is 22. Use proper function notation to express this. *We would have $22 = k(3)$.*

5 pts (c) Use proper function notation to express the change in output from k as the input values increase from -0.2 to 1.4. *We have $\Delta m = k(1.4) - k(-0.2)$.*

5 pts (d) Use proper function notation to express the average rate of change for the function k as as the input values increase from -0.2 to 1.4.

$$
\frac{\Delta m}{\Delta n} = \frac{k(1.4) - k(-0.2)}{1.4 + 0.2}
$$

10 pts 3. What is the domain of the function h defined by the output formula $u = h(t) = \sqrt{30 - 2t}$? You must show your work for full credit.

Solution. The function will be undefined when $30 - 2t < 0$, and this occurs when $15 < t$. The domain of the function will therefore be all values of t such that $t \leq 15$.

12 pts   4. The volume V of a sphere (in cubic inches) is related to the radius r of the sphere (in inches) according to the rule

$$V = h(r) = \frac{4\pi r^3}{3}$$

Identify all parts of this rule by matching them to the appropriate term.

____**C**____ r ____**E**____ $h(r)$

____**D**____ h ____**A**____ $h(r) = \frac{4\pi r^3}{3}$

(a) Output formula   (b) Constant Rate   (c) Input variable

(d) Function name   (e) Output   (f) Domain

5. Use the function defined in Problem 4 to answer the following questions.

6 pts   (a) What is the volume of the sphere when the radius is 2.5 inches? You must show your work for full credit.

Solution. The volume will be

$$h(2.5) = \frac{4\pi(2.5 \text{ in})^3}{3} \approx 65.45 \text{ in}^3$$

6 pts   (b) To the nearest hundredth, what is the radius of the sphere when its volume is 53 cubic inches? You must show your work for full credit.

Solution. The radius will be the solution to the equation $53 = h(r)$. Now, we know

$$53 = h(r) \implies 53 = \frac{4\pi r^3}{3}$$
$$\implies 39.75 = \pi r^3$$
$$\implies 12.65 \approx r^3$$
$$\implies 2.33 \approx r$$

6. Use the table below to determine the value of each expression. If it is not possible, explain why.

x	-1.0	0.0	0.5	1.6	2.0	2.2
$f(x)$	1.0	-2.2	0.0	0.5	2.2	3.0
$g(x)$	1.6	2.0	0.0	-1.0	3.0	0.5

5 pts   (a) $g(f^{-1}(1.0))$ *Not possible because f has no inverse*

5 pts   (b) $f(f(0.5)) = f(0.0) = -2.2$

5 pts   (c) $f(g^{-1}(3.0)) = f(2.0) = 2.2$

10 pts   7. Discount Car Rental charges Monique $15.00 to rent a car and charges $0.15 per mile that Monique drives the car.

(a) Construct the output formula for the function f that gives the cost C in dollars for Monique to rent a car in terms of the number m of miles she has driven. You must use proper function notation for full credit.

**Solution.** The output formula is $C = f(m) = 15.00 + 0.15m$.

(b) Construct the output formula for the function g that gives the number m of miles Monique has driven in terms of the cost C in dollars for Monique to rent a car. You must use proper function notation for full credit.

Solution. The function we seek will be the inverse of the function f. Now,

$$C = 15 + 0.15m \implies \frac{C - 15}{0.15} = m$$

Consequently, the desired function is $m = g(C) = \dfrac{C - 15}{0.15}$.

8. The graph of the function f is given below. Use these graphs to answer the questions posed.
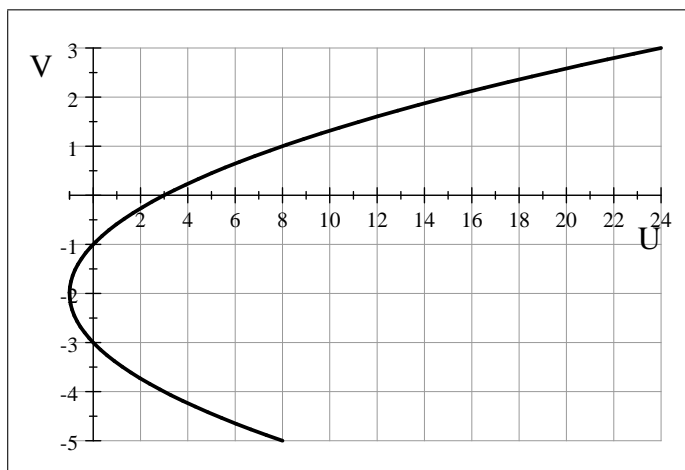
Graph of function f

5 pts   (a) What is the input variable for the function f and how do you know?

Solution. The graph fails the vertical line test, so it cannot represent V as a function of U. The graph does pass the horizontal line test and therefore represents U as a function of V.

6 pts   (b) Determine all of the input values that will produce an output value of 3 from the function f.

Solution. We want to solve the equation $3 = f(V)$ since V must be the input variable. The vertical line $U = 3$ crosses the graph of f twice, namely at the points $(3, 0)$ and $(3, -4)$. Therefore, there are two solutions to the equation, namely $V = 0$ and $V = -4$.

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