

# MATH 1730 EXAM III

(Version 2)

100 points

NAME: \_\_\_\_\_

Please place the letter of your selection in the blank provided. These questions are worth five points each.

- 5 pts 1.   **B**   If the table below gives some values for the number  $N$  of customers buying shoes from ShoeTunes  $t$  years after the company was founded, what is the approximate growth (or decay) factor for  $N$ ?

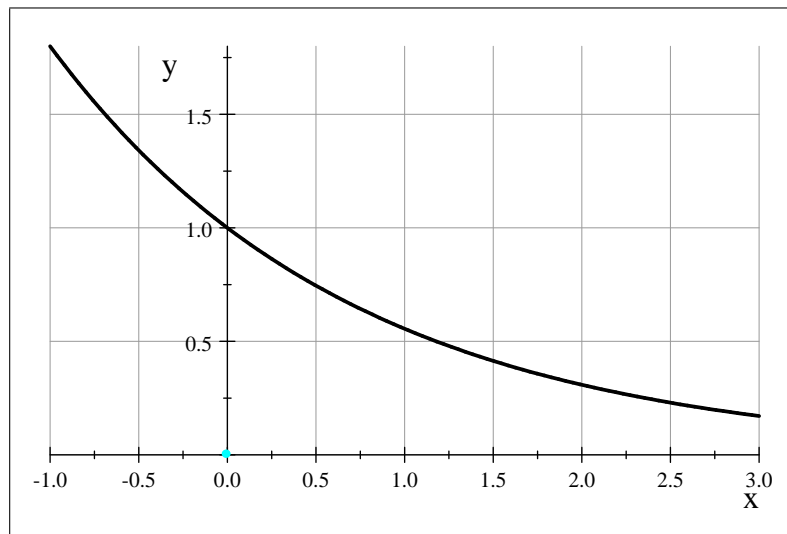
$t$	1	2	3
$N$	5,985	2,388	953

- (a)  $b \approx 1.399$                       (b)  $b \approx 0.399$   
(c)  $b \approx 1.414$                       (d)  $b \approx 0.714$   
(e)  $b \approx 1.60$

- 5 pts 2.   **C**   If you deposit \$3,000 into an account that pays 2% APR compounded weekly, which of the following expressions is the annual percent yield (APY) in decimal form for this account?

- (a)  $3000 \left(1 + \frac{0.02}{52}\right)^{52}$                       (b) 0.02  
(c)  $\left(1 + \frac{0.02}{52}\right)^{52} - 1$                       (d)  $\frac{0.02}{52}$   
(e)  $1 + \frac{0.02}{52}$

- 5 pts 3.   **C**   The figure below shows the graph of an exponential function  $f(x) = b^x$ . Based on this graph, the solution to the equation  $1.5 = b^x$  is



- (a)  $x \approx 0.40$                       (b)  $x \approx 3.00$   
(c)  $x \approx -0.60$                       (d)  $x \approx 1.40$   
(e)  $x \approx 1.10$

- 5 pts 4.     **E**     If we rewrite the expression  $\log_2 x - \log_2 9$  so that it is a single logarithm, we have
- (a)  $\log_2(9x)$  (b)  $9 \log_2(x)$   
(c)  $\log_2(9^x)$  (d)  $\log_2(x^9)$   
(e)  $\log_2\left(\frac{x}{9}\right)$
- 5 pts 5.     **B**     Jose plants a seven millimeter tall tomato seedling in his garden, and its height (in millimeters) increases 5% per day for the next thirty days. Which of the following functions gives the height  $H$  of the plant as a function of the number  $d$  of days since Jose planted it?
- (a)  $H(d) = 7 + .05d$  (b)  $H(d) = 7 \cdot 1.05^d$   
(c)  $H(d) = 7 \cdot (0.05)^d$  (d)  $H(d) = 7.35^d$   
(e)  $H(d) = 7 + 1.05d$
- 5 pts 6.     **E**     The salary of the vice president of the ShoeTunes corporation is given by  $P(t) = 60000 \cdot (1.03)^t$ , where  $P$  is measured in dollars, and  $t$  is the number of years since she was hired. Which of the following statements is true?
- (a) She will be receiving a 103% raise each year.  
(b) Her salary next year will be 3% of her salary this year.  
(c) Her salary increases by \$61,800 every year.  
(d) Her salary increases by \$1,800 every year.  
(e) Her salary next year will increase by 3% compared to her salary this year.
- 5 pts 7.     **A**     Consider the function whose output formula is  $N = f(t) = 175 \cdot 2^t$ . Which of the following formulas is the solution to the equation  $300 = f(t)$ ?
- (a)  $t = \log_2\left(\frac{300}{175}\right)$  (b)  $t = \log_{175}(300)$   
(c)  $t = \frac{\log(300)}{\log(175)}$  (d)  $t = \frac{\log_3(175)}{300}$   
(e)  $t = \frac{300}{175}$
- 5 pts 8.     **D**     Writing the equation  $\ln(2x - 1) = 6$  in exponential notation gives us
- (a)  $6 = \sqrt[6]{2x - 1}$  (b)  $2x - 1 = 10^6$   
(c)  $\ln = \sqrt[6]{2x - 1}$  (d)  $2x - 1 = e^6$   
(e)  $2x - 1 = \frac{6}{\ln}$

9. The table below gives some values for the number  $N$  of customers  $t$  years after the company ShoeTunes was founded.

$t$	1	2	3
$N$	5,985	2,388	953

- 5 pts (a) You determined the growth/decay factor in Problem 5 above. What is the percent change?

**Solution.** The percent change is  $0.399 - 1 = -0.601$  (a 60.1% decrease per year).

- 5 pts (b) What would the initial value be?

**Solution.** We know that  $5985 = a \cdot (0.399)^1$ , and this tells us that  $a \approx 15,000$ .

- 5 pts (c) Let  $f$  be the function that gives  $N$  as a function of  $t$ . What is the output formula for  $f$ ? Use proper function notation when writing your answer.

$$N = f(t) \approx 15000 \cdot 0.399^t$$

10. The owners of ShoeTunes invested \$45,000 in a CD that pays 3.8% APR, compounded *continuously*. Let  $V$  be the value of the CD in dollars  $t$  years after the initial investment.

- 5 pts (a) What is the formula that gives  $V$  as a function of  $t$ ?

$$V = f(t) = 45000 \cdot (e^{0.038})^t$$

- 10 pts (b) Use your formula from Part (a) to estimate how many years it will take for the value of the CD to double. You must show your work for full credit.

$$\begin{aligned} 90000 &= 45000 \cdot (e^{0.038})^t &\implies 2 &= (e^{0.038})^t \\ & &\implies \ln(2) &= 0.038t \\ & &\implies 18.24 &\approx t \end{aligned}$$

- 10 pts 11. Solve the equation  $\log_2(2x) + \log_2(x) - \log_2(9) = 2$  for  $x$ . Be sure to check your answers in the original equation.

**Solution.** First, we must rewrite the left-hand side of the equation as a single logarithm, and then rewrite the modified equation in exponential form. Observe

$$\begin{aligned} \log_2(2x) + \log_2(x) - \log_2(9) = 2 &\implies \log_2\left(\frac{2x \cdot x}{9}\right) = 2 \\ &\implies \log_2\left(\frac{2x^2}{9}\right) = 2 \\ &\implies \frac{2x^2}{9} = 2^2 \\ &\implies x^2 = 18 \\ &\implies x = \pm 3\sqrt{2} \end{aligned}$$

Since  $\log_2(x)$  is undefined when  $x = -3\sqrt{2}$ , only  $x = 3\sqrt{2}$  is a solution to the original equation.

- 10 pts 12. Use the laws of logarithms to write the expression below as a single logarithm.

$$4\log_4(1-a) - \log_4(a) + \log_4(7)$$

**Solution.** Since raising to a power must be done before multiplication, the reducing process must begin with the coefficients of the logarithms. Observe

$$\begin{aligned} 4\log_4(1-a) - \log_4(a) + \log_4(7) &= \log_4(1-a)^4 + \log_4(a^{-1}) + \log_4(7) \\ &= \log_4[(1-a)^4 \cdot a^{-1} \cdot 7] \\ &= \log_4\left[\frac{7(1-a)^4}{a}\right] \end{aligned}$$

10 pts 13. The amount of money in a bank account paying discretely compounded interest is given by the formula

$$P = g(t) = 2500 \cdot \left(1 + \frac{0.0254}{4}\right)^{4t}$$

where  $t$  represents the number of years passed since the initial deposit. Identify all parts of this rule by matching them to the appropriate term. Some terms will not be used.

<u>      <b>D</b>      </u> 0.0254	<u>      <b>C</b>      </u> 2500	<u>      <b>B</b>      </u> $\frac{0.0254}{4}$
<u>      <b>A</b>      </u> $1 + \frac{0.0254}{4}$	<u>      <b>F</b>      </u> $\left(1 + \frac{0.0254}{4}\right)^4$	

- (a) Per-quarter growth factor      (b) Per-quarter percent change      (c) Initial deposit  
(d) Annual percent change (APR)      (e) Annual percent yield (APY)      (f) Annual growth factor