Example 1. Challenger Deep

(a) James Cameron dived to a depth of 10898 m in the Challenger Deep. Estimate the pressure at that depth. (Assume the density of sea water is 1024 kg/m³.)

(b) The lung capacity of the average adult male is (according to Wikipedia) about 6 L. (i) Express this volume in both cubic centimeters and MKS units. (ii) At atmospheric pressure and room temperature (300 K), how many moles of gas molecules are present in James Cameron’s lungs? (Assume he is an average adult male.)

(c) If Cameron was silly enough to get outside his sub when submerged at a depth of 10898 m, what volume would his lungs occupy? (Assume that he is somehow able to hold the air in his lungs, and that the air remains at room temperature.) Express your answer in MKS units and cubic centimeters.
Example 2. Bubbles

A small bubble of gas (assumed to behave as an ideal gas) is formed at a depth beneath the surface of a calm lake where the pressure is $4.0 \times 10^5$ Pa. The bubble has a radius of 4.7 mm. The density of the water is 1010 kg/m$^3$, and its temperature is a constant 13 °C. We will assume that the bubble is spherical in shape. (The volume of a sphere is $\frac{4}{3}\pi r^3$.) Express ALL answers in SI units.

(a) At what depth below the surface of the lake did the bubble form?

(b) What is the value of the buoyant force acting on the bubble at the depth found in (a)?

(c) Approximately how many moles of gas molecules are contained in the bubble?

(d) As a result of the buoyant force acting on it, the bubble floats to the surface of the lake. What is the volume of the bubble just before it reaches the surface and pops?

(e) Just before the bubble pops at the surface of the lake, do you think that the buoyant force acting on it is less than, greater than, or the same as the value that you found in part (a) above? Explain your reasoning. (Do not calculate the buoyant force for this part.)

\[ P = P_{\text{top}} + \rho gh \]

\[ \therefore \; h = \frac{P - P_{\text{top}}}{\rho g} = \frac{4.0 \times 10^5 - 1.01 \times 10^5}{(1010)(9.8)} = \boxed{30.2 \text{ m}} \]

\[ F_B = m_k g = \rho V_{\text{depl}} g = \rho \frac{4}{3} \pi r^3 g = (1010)\left(\frac{4}{3}\pi\right)(0.0047^3)(9.8) = \boxed{4.3 \times 10^{-3} \text{ N}} \]

\[ PV = nRT \]

\[ n = \frac{PV}{RT} = \frac{(4 \times 10^5)\left(\frac{4}{3}\pi\right)(0.0047^3)}{(8.31)(286)} = \boxed{7.3 \times 10^{-5} \text{ mol}} \]

\[ PV = nRT \]

\[ V = \frac{nRT}{P} = \frac{(7.3 \times 10^{-5})(8.31)(286)}{1.0 \times 10^5} = \boxed{1.7 \times 10^{-6} \text{ m}^3} \]

\[ U_{\text{top}} < U_{\text{bottom}} \; \therefore \; F_B = \rho VG \; \text{so the buoyant force at the top must be larger than the value found in (a).} \]