Lecture 4

Freefall Motion

When an object is released near the earth’s surface, it starts falling towards the earth’s center as a result of the earth’s gravitational pull on the object. This is true whether the object is released from rest (dropped) or if it is thrown (up or down, sideways – whatever!). We will see a very different behavior of the object when it is thrown, however, if that object is a crumpled-up piece of paper or if it is a rock, even if they are thrown in exactly the same way. The reason for the difference in motion of these two objects after being thrown is that the presence of the air or a slight breeze can greatly affect the motion of the paper, while the rock moves in about the same way whether the air is present or not. (A rock falling through air and one falling through a vacuum show just about the same behavior. Indeed, a rock falling through vacuum and a crumpled-up piece of paper falling through vacuum fall in exactly the same way!)

If the motion of an object near the earth’s surface is virtually unaffected by the presence of the earth’s atmosphere (that is, air resistance hardly affects its motion at all), and the object’s motion is predominantly determined by the earth’s gravitational pull, then we say that the object is undergoing free-fall motion.

Any object undergoing free-fall motion at the earth’s surface has an acceleration which has a direction pointing directly towards the center of the earth. This very special acceleration (to us here on Earth, at least!) is called the acceleration due to gravity, and is given the special symbol $\vec{g}$. The magnitude of the acceleration due to gravity at the earth’s surface is

$$g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$$

Please be careful of the distinction between $\vec{g}$ and $g$; the former is a vector, and it always points towards the center of the earth. The latter is the magnitude of the acceleration vector – it is ALWAYS a positive quantity: $g = +9.8 \text{ m/s}^2$ ALWAYS.

The only thing new in this lecture is the fact that, under the conditions for free-fall motion described above, we now automatically know the acceleration of an object. (Note that this acceleration does not depend on how heavy the object is! Does this make any sense to you? It may or may not now, but it should later on in this course.) Since this is a constant acceleration, the kinematic equations must still apply. We can therefore apply our kinematic expertise to solving free-fall problems!

The kinematic equations (again)

For your convenience in solving the free-fall problems in this lecture, the table of the four 1-D kinematic equations of motion from Lecture 3 has been duplicated below. Remember that this set of equations is valid for describing the motion of an object only if the acceleration of the object is a constant.
4.1 Freefall examples

Example 4.1
You throw a rock vertically upward. After leaving your hand, it moves upwards a distance of 34 ft before stopping and falling back down towards the ground. (a) How fast was the rock moving the instant it left your hand? (b) How long is it before you catch the rock?

[Answers: (a) 47 ft/s (b) 2.9 s]

Solution: You want to watch out for the units that are being used in these problems. Note that the distance is given in units of feet. This means that the value of the acceleration due to gravity, \( g \), that we want to use in this problem is \( g = 32 \text{ ft/s}^2 \).

(a) How fast was the rock moving the instant it left your hand?

This question is also sometimes posed in the following way: “What was the initial speed of the rock?”, or “How fast was the rock moving when you threw it?”. It is important to keep in mind that, as long as we want to describe the motion of an object and we wish to use the kinematic equations to do it, the acceleration of the object (change in velocity over the corresponding change in time) must be a constant. When your hand is still in contact with the rock and is pushing it upwards, the rock does not have a constant acceleration, and you can’t apply the kinematic equations. Since this is a kinematics problem, it is implied that the initial point of the rock’s motion is the instant it left your hand (since your hand is no longer exerting a force on it) and, wherever you choose the final point, it must be before anything else acts on the rock (such as the ground or your hand again).

Now – on with the solution. We first (of course) draw a picture and list the kinematic variables. Since we are dealing with the vertical direction, we will use the variable \( y \) instead of \( x \). Note that we are calling \( y = 0 \) at the point where the rock leaves your hand (the initial point).
There are a couple of important notes here. First, since we are given information about the top of the motion, we pick this as the final point for our equations. (We have to use points for which we have information!) Second, note that, since we chose upwards to be the positive-y direction, and since the acceleration due to gravity points downwards, it follows that the y-component of acceleration must be negative. (Watch out for this!) We wish to solve for the initial velocity; we don’t know the time, $t$, so we pick the equation with $t$ missing:

$$v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$$

Substituting in zero values and solving algebraically for the desired unknown, we get that

$$0 = v_{iy}^2 + 2a_yy_f,$$

or

$$v_{iy} = \pm \sqrt{-2a_yy_f} = \pm 47 \text{ ft/s}.$$ 

Note that we get two solutions, one positive and one negative, corresponding to upward and downward velocities (up when you threw the stone, down when it was falling back to earth). The initial velocity must be positive (because of the way we chose the positive-y direction), so our final answer is $v_{iy} = 47 \text{ ft/s}$. 

(b) How long is it before you catch the rock?

With just a bit of thought, this part is now very easy. The only unknown in our kinematic variables above is $t$. We can therefore use any of the remaining kinematic equations to solve for $t$. We’ll pick the easiest one:

$$v_{fy} = v_{iy} + a_yt$$

$$0 = v_{iy} + a_yt$$

$$t = -\frac{v_{iy}}{a_y} = 1.46 \text{ s}.$$ 

But, this is the time to go from your hand up to the highest point. What was asked for was the time from when you threw the rock until when it reached your hand again. The time it takes for the rock to go up (as it slows down) is the same time that it takes the rock to fall back down again (as it speeds up). (Does this make sense to you? Can you prove this using the kinematic equations?) Therefore, the total time for the rock to go up and down is just twice the time we calculated above. We therefore have that the total time is $t = 2.9 \text{ s}$. □ 

Example 4.2

A diver springs upward with an initial speed of 2.3 m/s from a 3.0-m board. (That is, the diving board is 3.0 m above the water’s surface.)

(a) Find the y-component of her velocity when she strikes the water.

(b) What is her greatest height above the water during her dive?
(c) What was her acceleration at the highest point?

[Answers: (a) 8.0 m/s downward (b) 3.3 m (c) 9.8 m/s² downward]

**Solution:** Note that the distance we are given and the question about the height (part b) are both referenced from the water’s surface. We therefore take the water’s surface to be the \( y = 0 \) level, and call the positive-y direction upward.

![Diagram of diver]  
(a) Find the y-component of her velocity when she strikes the water.

\[ y_i = 3.0 \text{ m}, \quad y_f = 0, \quad v_{iy} = 2.3 \text{ m/s}, \quad v_{fy} = ?, \quad a_y = -9.8 \text{ m/s}^2, \quad t =? \]

Note that, in this case, the value of the initial position is *not* zero. This is simply a result of the way we chose the origin and the positive-y direction. You are free to do it any way you want, as long as you show your choice on your diagram and then stick with what you have chosen. Since we neither know nor want \( t \) at this point, we choose the equation with \( t \) missing:

\[
v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)
\]

\[
= v_{iy}^2 - 2a_yy_i.
\]

Thus,

\[
v_{fy} = \pm \sqrt{v_{iy}^2 - 2a_yy_i} = \pm 8.0 \text{ m/s}.
\]

We know that the final velocity is pointing downwards in the negative direction, so our answer is \( v_{fy} = -8.0 \text{ m/s} \).

(b) What is her greatest height above the water during her dive?

Note that in part (a) the initial point \( y_i \) was at the board and the final point \( y_f \) was at the water’s surface. Neither of these two points will address this question of where the diver was at her highest position above the water’s surface. We thus have to change one or the other of our two points for the kinematic equations. Let’s keep the initial point the same (since this information was given to us; if we somehow goofed in our solution above, it won’t affect our answer here . . . ), but change the final point to where the diver reaches her highest point. An important question at this point is how are we going to communicate to the kinematic equations that the diver is at her highest point? Do you see what is special about the highest point of the diver’s motion? What happens to her speed at the highest point (and only at the highest point)? See if you were correct in the kinematic information below.

\[ y_i = 3.0 \text{ m}, \quad y_f =?, \quad v_{iy} = 2.3 \text{ m/s}, \quad v_{fy} = 0, \quad a_y = -9.8 \text{ m/s}^2, \quad t =? \]
The key idea about the highest point of free-fall motion is that the vertical component of the object’s velocity must be zero, since this is the point where the object has instantaneously stopped and is turning around to start falling back down towards the earth. Having realized this, the answer to the question is now straightforward:

\[ v_f^2 = v_i^2 + 2a_y(y_f - y_i) \]
\[ 0 = v_i^2 + 2a_y(y_f - y_i) \]
\[ y_f = y_i - \frac{v_i^2}{2a_y} = 3.3 \text{ m}. \]

The diver thus only goes 0.3 m or 30 cm above her starting position before starting to fall back down again!

(c) What was her acceleration at the highest point?

This is one of the most commonly missed questions in this course! The most common (and incorrect!) answer is zero. When people give this answer, you know that they are thinking of the vertical component of velocity at the highest point, not the acceleration! For any point of the motion during free fall, the acceleration is always the acceleration due to gravity, \( \vec{g} = 9.8 \text{ m/s}^2 \) (downward)!

Example 4.3

You are standing at the foot of a cliff holding a helium balloon. You release the balloon from the base of the cliff, and it immediately moves upwards with a constant velocity. (This is an approximation – in reality the balloon may not have a constant velocity.) The instant you release the balloon, your evil twin drops a jagged-edged rock from the top of the cliff a distance of 12 m directly above the balloon. A time 1.2 s after releasing the balloon, you hear the balloon pop as it’s hit by the rock. What was the constant speed of the balloon?

[Answer: 4.1 m/s (Let’s see how good you are! Can you get this answer without looking at the solution? This is another pretty tricky problem – you’re doing well even if you can only get the picture drawn and the kinematic quantities written down correctly!)]

Solution: As usual, we start off with a drawing and a listing of the kinematic quantities. As in Example 3.4, we have two objects to keep track of (in Ex. 3.4, a car and a truck; in this problem, a rock and a balloon). Again, the key to the solution is seeing which kinematic quantities are common to both objects. Do you see what they are?
Balloon: \( y_{i,b} = 0 \quad y_f = ? \quad v_b = ? \) (constant) \( a_{y,b} = 0 \quad t = 1.2 \, \text{s} \)

Rock: \( y_{i,r} = 12 \, \text{m} \quad y_f = ? \quad v_{y,r} = 0 \quad v_{fy,r} = ? \quad a_{y,r} = -9.8 \, \text{m/s}^2 \quad t = 1.2 \, \text{s} \)

Note that we have called \( y = 0 \) at the initial height of the balloon, and the positive-y direction upward. Also note that the symbols for \( t \) and \( y_f \) have no subscripts for the balloon or rock since these variable values are the same for both of the objects. (Did you get it right?) The acceleration of the balloon is zero since we are told that the velocity of the balloon is a constant (since its speed is a constant, and it is only going upward).

Note that we only have two question marks for the rock data. This means that we can solve for any variable that we want for the rock. Let’s solve for \( y_f \), since this will give us more information for the balloon’s motion:

\[
y_f = y_{i,r} + v_{y,r} t + \frac{1}{2} a_{y,r} t^2 = 4.9 \, \text{m}.
\]

Since the speed of the balloon is constant, it must be the same as the average speed, which means that it must satisfy the equation

\[
v_b = \frac{y_f - y_{i,b}}{t} = \frac{y_f}{t}
\]

Substituting in the value for \( y_f \) found above, we then get that

\[
v_b = \frac{y_f}{t} = 4.1 \, \text{m/s}
\]

Note that this problem can be made even more difficult, but also more accurate, by noting that the speed of sound in air is 343 m/s. You do not hear the balloon pop the instant that it happens – the sound actually has to travel down from the height at which the balloon pops above your head and reach your ear. If you take this effect into account, you will come up with a slightly different answer (since the 1.2 s now also includes the time for the sound to travel back down to your head). Do you think that this would make the new answer for the balloon speed larger or smaller than the value obtained above? See if you can solve this problem taking into account the speed of sound if you want to see how really good you are! □
4.1. FREEFALL EXAMPLES

Homework

Warm-up exercises

1. A diver springs vertically upwards from a 4.5-m board with an initial speed of 4.2 m/s. (a) What is the maximum height that she reaches above the water? (b) What is her acceleration when she is at this maximum height?

2. Let’s say that you can throw a baseball as fast as 14 m/s. You throw the ball at this speed straight up into the air. How high does the ball go as measured from the point that you released it from your hand?

Some standards

3. An innocent bystander is standing-by on the sidewalk by a building waiting for a taxi. Little Jimmy Quagmyre, the neighborhood spoiled brat and all-around obnoxious little monster, is leaning out of the third-story window holding a balloon filled with molasses (don’t you just love him?!). He drops the balloon onto the bystander. How fast is the balloon traveling when it hits the bystander on the head if Jimmy is 11 m above the sidewalk and the bystander is 2-m tall?

4. That’s it – Sally has really gone too far this time, and Billy just can’t take it anymore. Billy runs over to Sally, grabs the damn doll (Princess Beutifica – yeah, right; give me a break!), leans out of the window and throws the beautiful princess straight down towards the ground with a speed of 2.9 m/s. If the window is 12 m above the ground, (a) how long does it take the princess to hit the ground, and (b) how fast is she moving when she bites the dust?

5. Fern Thimblehead leans out of a window and throws a gold ball straight up into the air with a speed of 16 m/s. The ball hits the ground 3.6 s later. (a) How high is the window above ground level? (b) How fast is the ball moving when it hits the ground?

So, you think you’re pretty good…?

6. Pfft Drrpt, a Blnng from the planet Glrrb, loves to toss his Flllt into the air (at least what passes for air on Glrrb). On one toss, Pfft tosses his Flllt vertically upwards with an initial speed of 22 m/s from an initial height of 2.4 m above the surface (the average height of a Glrrbian Blnng). The Flllt hits the surface 5.8 s later. What is the acceleration due to gravity at the surface of the planet Glrrb? (Neglect any air resistance and assume the Flllt is in freefall.)

7. The evil criminal has kidnapped the Duchess, stolen the jewels, and taken both to his cheap hotel room to think out what evil deed he can do next, unaware of the fact that Superboy! is at that very moment on top of the hotel building. When the criminal leans out of his window to see if the coast is clear, Superboy! cleverly drops a 2-ton boulder onto his head, thereby saving the day! Superboy! is 20.0 m above the ground when he drops the boulder, and the evil criminal is 11.0 m above the ground as he leans out of the window. (a) What is the speed of the boulder as it hits the evil criminal’s head? (b) How long did it take for the boulder to reach the evil criminal’s head after it was dropped? (c) What was the average speed of the boulder from when it was dropped until when it hit the evil criminal’s head?
Answers

1. (a) 5.4 m  (b) 9.8 m/s² downward

2. 10 m

3. 13.3 m/s

4. (a) 1.3 s   (b) 16 m/s

5. (a) 5.9 m   (b) 19 m/s

6. 7.7 m/s² downward

7. (a) 13.3 m/s   (b) 1.36 s   (c) 6.62 m/s