Lec 10I Homework

Problem 10.2 Calculate the distance $r_0$ between the K$^+$ and Cl$^-$ ions in KCl, assuming that each ion occupies a cubic volume of side $r_0$. The molar mass of KCl is 74.55 g/mol and its density is 1.984 g/cm$^3$.

Let $n =$ # density of ions in KCl

$$n = \frac{\# \text{ions}}{\text{volume/ion}}$$

Also, $\# \text{ions} = \frac{V}{V_{\text{ion}}}$

Now $V_{\text{ion}} = r_0^3$

$$\therefore n = \frac{V}{V_{\text{ion}}} \times \frac{1}{V}$$

$$= \frac{1}{r_0^3}$$

$$\therefore r_0 = \left(\frac{1}{n}\right)^{\frac{1}{3}}$$

To find $n$:

Molar mass = 74.55 g

$\rho = \frac{1.984 \text{ g}}{\text{cm}^3}$

$\therefore$ There are $\frac{1}{74.55} \text{ moles} \times \frac{1.984 \text{ g}}{\text{cm}^3}$

$$= 2.661 \times 10^{-2} \text{ moles/cm}^3$$

This means there are $N_A \times 2.661 \times 10^{-2}$

$$= 6.022 \times 10^{23} \times 2.661 \times 10^{-2}$$

$$= 1.603 \times 10^{22} \text{ molecules/cm}^3$$

Since there are two ions per molecule, we have $n = 2 \times 1.603 \times 10^{22} \text{ ions/cm}^3$

$$= 3.206 \times 10^{22} \text{ ions/cm}^3$$

$$\therefore r_0 = \left(\frac{1}{n}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1}{3.206 \times 10^{22}}\right)^{\frac{1}{3}}$$

$$= 3.15 \times 10^{-8} \text{ cm}$$

$$= 0.315 \text{ nm}$$
Problem 10.11 Find (a) the current density and (b) the drift velocity if there is a current of 1 mA in a No. 14 copper wire. (The diameter of No. 14 wire, which is often used in household wiring, is 0.064 in = 0.163 cm.)

(a) \[ I = 1 \text{ mA} = 1 \times 10^{-3} \text{ A} \]
\[ r = \frac{0.163}{2} \text{ cm} \]
\[ \rho = \frac{I}{A} \]
\[ = \frac{1 \times 10^{-3}}{\pi \times \left( \frac{0.163 \times 10^{-2}}{2} \right)^2} \]
\[ = \frac{479}{\text{A/m}^2} \]
\[ = 479 \times 10^3 \frac{\text{mA}}{\text{m}^2} \text{ cm} \]

(b) \[ I = nAv_e \quad \text{(from lecture notes)} \]
\[ n = \# \text{density of e}^- \]
\[ = 8.47 \times 10^{28} \frac{\text{m}^{-3}}{} \quad \text{(from Table 10-3)} \]
\[ V_d = \frac{I}{An} \]
\[ = \frac{J}{ne} \]
\[ = \frac{479}{8.47 \times 10^{28} \times 1.6 \times 10^{-19}} \]
\[ = 3.54 \times 10^8 \text{ m/s} \]
HW 10-II Solutions
Problem 10.13 Calculate the number density of free electrons in (a) Ag ($\rho = 10.5$ g/cm$^3$) and (b) Au ($\rho = 19.3$ g/cm$^3$), assuming one free electron per atom, and compare your results with the values listed in Table 10-3.

(a) Ag, $\rho = 10.5 \frac{g}{cm^3}$, molar mass = $107.87 \frac{g}{mol}$ (from periodic table).

$$n = \frac{\text{# atoms}}{m^3} = \frac{1}{\text{mol}} \times \frac{10.5 \frac{g}{cm^3}}{\frac{g}{mol}} \times \frac{3 \times (\frac{100 \text{ cm}^3}{1 \text{ m}})}{1 \text{ m}^3} \times N_A$$

$$= 5.86 \times 10^{28} \frac{\text{atoms}}{m^3} \quad \text{same as table 10-3.}$$

(b) Au, $\rho = 19.3 \frac{g}{cm^3}$, molar mass = $196.97 \frac{g}{mol}$

$$n = \frac{\text{# atoms}}{m^3} = \frac{1}{196.97 \frac{g}{mol}} \times 19.3 \frac{g}{cm^3} \times \frac{3 \times (\frac{100 \text{ cm}^3}{1 \text{ m}})}{1 \text{ m}^3} \times N_A$$

$$= 5.90 \times 10^{28} \frac{\text{atoms}}{m^3} \quad \text{same as table 10-3.}$$
Lec 10-II Homework

Problem 10.18  Compute (a) the Fermi energy and (b) the Fermi temperature for silver and for iron and compare your results with the corresponding values in Table 10-3.

(a) \[ E_F = \frac{\hbar^2}{2m} \left( \frac{3}{8\pi N} \right)^{2/3} \]

Eq 10-20,

\[ = \frac{\hbar^2}{2m} n^{2/3} \left( \frac{3}{8\pi} \right)^{1/3} \]

where \( n = \frac{N}{V} = \# \text{ density} \)

\[ m = m_e = 9.11 \times 10^{-31} \text{ kg} \]

\[ n = 5.86 \times 10^{28} \text{ m}^{-3} \quad \text{for Ag} \]

\[ n = 17.0 \times 10^{28} \text{ m}^{-3} \quad \text{for Fe} \]

From Table 10-3,

\[ E_F = 8.91 \times 10^{-19} \text{ J} = 5.51 \text{ eV} \quad \text{for Ag} \]

\[ (\text{c.f. } 5.53 \text{ eV in Table 10-3. Close}) \]

\[ E_F = 1.79 \times 10^{-19} \text{ J} = 11.20 \text{ eV} \quad \text{for Fe} \]

\[ (\text{c.f. } 11.2 \text{ eV in Table 10-3.}) \]

(b) \[ T_F = \frac{E_F}{k} \]

Eq 10-23

Using \( k = 8.617 \times 10^{-5} \text{ eV/K} \), we have

\[ T_F = \frac{64\,000 \, K}{64\,000 \, K} \quad \text{for Ag} \]

\[ (6.4 \times 10^4 \, K \text{ in Table 10-3}) \]

\[ T_F = \frac{13 \times 10^4 \, K}{13 \times 10^4 \, K} \quad \text{for Fe} \]

\[ (13 \times 10^4 \, K \text{ in Table 10-3}) \]
**Extra Problem** Consider a general vector having the direction angles $\alpha$, $\beta$, and $\gamma$, as described in lecture. Prove the following important relation between these angles:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

*Hint:* This is reasonably straightforward if you take the dot product of the vector with each of the three unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$. Think about what these dot products mean.

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**Method 1**

$$\vec{v} \cdot \hat{x} = v_x$$

Also $\vec{v} \cdot \hat{x} = v \cos \alpha$

$$\therefore v_x = v \cos \alpha$$

Similarly,

$$v_y = v \cos \beta \quad \text{and} \quad v_z = v \cos \gamma$$

Now $v_x^2 + v_y^2 + v_z^2 = v^2$

$$v^2 (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = v^2$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$
HW 10-III Solutions
**Extra problem 1.** *Diamagnetism* occurs when an electron orbiting a nucleus responds to an external magnetic field. The electron’s orbit changes in such a way (see lecture notes) as to cause a change in the magnetic moment that is in the opposite direction to the external B-field. All materials exhibit diamagnetism, but it is such a weak effect ($|\chi| < 10^{-5}$) that it is not usually observable. The susceptibility $\chi$ is also negative, since the induced magnetism is in the opposite direction to the external B field.

*Paramagnetism* is associated with the intrinsic spin of electrons. It is only exhibited by atoms which have odd number of electrons, since in atoms with even numbers, the electrons are paired up and the spins cancel. In the presence of an external B field, the intrinsic electron magnetic moments align with the field and so the induced magnetism is in the same direction as the external field ($\chi > 0$). For paramagnetic materials, $\chi \sim 10^{-4}$.
Extra problem 2.

(a) \[ U = -\vec{\mu} \cdot \vec{B} = \mu B \cos \theta \]

\[ U_{\text{min}} = -\mu B \quad \text{when } \theta = 0 \quad (\text{parallel}) \]
\[ U_{\text{max}} = \mu B \quad \text{when } \theta = 180 \quad (\text{antiparallel}) \]

(b) For \( B = 2 \) T and \( \mu = 9.3 \times 10^{-24} \) J/T we have

\[ U_{\text{min}} = -1.86 \times 10^{-23} \text{ J} \]
\[ U_{\text{max}} = 1.86 \times 10^{-23} \text{ J} \]

(c) \[ \bar{\mu} = \mu \frac{e^{\mu B_{\text{ext}}/kT} - e^{-\mu B_{\text{ext}}/kT}}{e^{\mu B_{\text{ext}}/kT} + e^{-\mu B_{\text{ext}}/kT}} \]

As \( B \to \infty \) the terms with negative exponents go to zero. Thus as \( B \to \infty \) we have

\[ \bar{\mu} = \mu \frac{e^{\mu B_{\text{ext}}/kT}}{e^{\mu B_{\text{ext}}/kT} + e^{-\mu B_{\text{ext}}/kT}} = \mu \]

Then

\[ M_{\text{max}} = n\bar{\mu} = 5 \times 10^{28} \times 9.3 \times 10^{-24} = 4.65 \times 10^5 \text{ T.A}^2/N \]
HW 10-IV Solutions
\[ E_{ph} = \frac{hc}{\lambda} \implies \lambda = \frac{hc}{E_{ph}} \]

Use \( c = 3 \times 10^8 \text{ m/s} \)
\( h = 4.136 \times 10^{-15} \text{ eV s} \)

Si \( \lambda = 1088 \text{ nm} \)
Ge \( \lambda = 1723 \text{ nm} \)
Diamond \( \lambda = 577 \text{ nm} \).

10.28
(a) From 10.27, all photons with energy above 0.72 eV will be absorbed by Ge.
This means all light with wavelength less than 1723 nm will be absorbed.
\( \implies \) No visible light will be transmitted.
(b) \( E_g = 3.6 \text{ eV} \implies \lambda = 345 \text{ nm} \) i.e. photons with wavelength larger than 345 nm will not be absorbed.
Range of wavelengths in visible that will be transmitted is
\[
\begin{array}{c}
400 \text{ nm} \\
450 \text{ nm - 700 nm}
\end{array}
\]

10.29
(a) \( \lambda = 3.35 \mu \text{m} \)
\( E_g = E_{ph} = \frac{hc}{\lambda} = 0.37 \text{ eV} \).
(b) \( K_T = 0.37 \text{ eV} \)
\( K = 8.617 \times 10^{-5} \frac{\text{eV}}{K} \)
\( T = \frac{0.37}{K} \approx 4300 \text{ K} \)
\[
\rho = 2.33 \, \text{g cm}^{-3}
\]
(a) Find \(N_a\), the total number of Si atoms in the crystal.

\[
\rho = \frac{m}{V}
\]
Let \(n\) = moles of Si atoms.

Then \(m = nM_{Si} = \frac{N}{N_h} M_{Si}\).

\[
\therefore \rho = \frac{nM_{Si}}{N_h V}
\]

\[
\therefore N = \rho V = \frac{2.33 \times 6.02 \times 10^{23} \times (100 \times 10^{-7})^3}{28.09} = 5.0 \times 10^3 \text{ atoms}
\]

(b) \(E = 1.3 \, \text{eV} \), width of conduction band.

no. of states = 4N.

\[
\therefore \text{Energy spacing} = \frac{E}{4N} = \frac{1.3}{4 \times 10^3} = 6.5 \times 10^{-8} \, \text{eV}
\]

(a) Si doped with Al \(\Rightarrow\) p-type silicon \((\text{Al is an acceptor})\).

(b) Si doped with P \(\Rightarrow\) n-type silicon \((\text{P is a donor})\).

\[
\frac{KT}{e} = 0.01 \, \text{eV}
\]

\[
T = \frac{0.01 \, \text{eV}}{8.617 \times 10^{-5}} = 11.6 \, \text{K}
\]
(a) Intrinsic Si

(b) Si doped with gallium

Gallium is an $\alpha^-$ acceptor, so this is p-type silicon.

(c) Si doped with arsenic

Arsenic is an $\varepsilon$ donor, so this is n-type silicon.
HW 10-V Solutions
Problem 10-38  Compute the fractional change in the current through a pn junction diode when the forward bias is changed from 0.1 V to 0.2 V.

Solution.  Ideal diode equation:

\[ I = I_0 (e^{V_b/kT} - 1) \]

Let \( V_1 = 0.1 \) V and \( V_2 = 0.2 \) V. Then we have

\[ I_1 = I_0 (e^{V_1/kT} - 1) \]

and

\[ I_2 = I_0 (e^{V_2/kT} - 1) \]

We want the fraction \( \frac{\Delta I}{I_1} \), where \( \Delta I = I_2 - I_1 \). Assuming room temperature (\( T = 300 \) K) and plugging in the numbers gives

\[ \frac{\Delta I}{I_1} = 47.9 \]
Problem 10-40  When light of wavelength no larger than 484 nm illuminates a CdS solar cell, the cell produces current. Determine the energy gap in CdS.

Solution.  A photon requires a minimum energy of $E_{\text{ph}} = E_g$ in order to be absorbed. The minimum energy corresponds to the largest wavelength that can be absorbed, in this case 494 nm. Thus

$$E_g = E_{\text{ph}} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{494 \times 10^{-9}} = 4.11 \times 10^{-19} \text{ J} = 2.57 \text{ eV}$$