NUCLEAR REACTIONS

Lecture 11-VI
Radioactive decay – a nucleus spontaneously decays. The only particle present before the decay is the parent nucleus.

Nuclear reaction – an incident particle interacts with a target nucleus, resulting in two or more reaction products.

The generic reaction

\[ a + A \rightarrow B + b \]
“Chemical reaction” notation

\[ a + A \rightarrow B + b \]

Common shorthand for nuclear reactions

\[ A(a, b)B \]

**Examples**

Write the following reactions using the shorthand notation. *(Note: conservation laws)*

\[
\begin{align*}
\frac{4}{2}He + \frac{14}{7}N &\rightarrow \frac{1}{1}H + \frac{17}{8}O \\
\frac{1}{1}H + \frac{9}{4}Be &\rightarrow \frac{9}{5}B + \frac{1}{0}n
\end{align*}
\]

First documented nuclear reaction (Rutherford, 1919)
A simple model of nuclear reactions proposed by Bohr (1936) in which an incident projectile that bombards a nucleus undergoes a strong interaction with all of the constituents of that nucleus, resulting in the incident particle’s energy being equally shared by all of the nucleons. The compound nucleus thus formed would then decay in a manner independent of the way in which it was formed.

Current models of nuclear reactions are more complicated than this, involving scattering of the incident particle’s wavefunction by the nuclear potential energy function, but some aspects of the compound nucleus are still useful in a conceptual understanding of nuclear reactions.
The Compound Nucleus

Examples

\[
\begin{align*}
\text{entrance channels} & \quad \text{compound nucleus} & \quad \text{exit channels} \\
10^5 \text{B} + \alpha & \rightarrow 14^7 \text{N}^* & \rightarrow 12^6 \text{C} + \text{d} \\
13^7 \text{N} + \text{n} & \rightarrow 14^7 \text{N}^* & \rightarrow 10^5 \text{B} + \alpha \\
14^7 \text{N} + \gamma & \rightarrow 14^7 \text{N}^* & \rightarrow 14^7 \text{N} + \gamma \\
12^6 \text{C} + \text{d} & \rightarrow 14^7 \text{N}^* & \rightarrow 13^7 \text{N} + \text{n}
\end{align*}
\]
Cross-sections for Reactions

The cross-section for a given reaction gives a sense of how probable that reaction is.

\[ \sigma \equiv \frac{N}{I(nA\Delta x)} \equiv \frac{R}{I} \]

- \( I \) = number of incident projectiles per unit area per unit time (incident intensity)
- \( N \) = number of reactions per unit time
- \( n \) = number of target nuclei per unit volume in the target
- \( A \) = area of target bombarded by the incident particles
- \( \Delta x \) = target thickness
- \( R \) = nucleus reaction rate = number of reactions per unit time per nucleus

Units: 1 barn = \( 10^{-28} \) m\(^2\)

Image courtesy Dr. Montemayor's Modern II website.
Differential Cross-sections

\[ dN = \text{number of light reaction products scattered into a solid angle } d\Omega \text{ at the polar angle } \theta \text{ and the azimuthal angle } \phi \text{ during the time } t \]

Notations for differential cross-sections:

\[
\sigma(\theta, \phi) = \frac{d^2\sigma}{d\theta \, d\phi} = \frac{d\sigma}{d\Omega} = \frac{dN/d\Omega}{I(nA\Delta x)}
\]

The cross-section:

\[
\sigma = \int \frac{d\sigma}{d\Omega} \, d\Omega \quad \text{all space}
\]

Image courtesy Dr. Montemayor's Modern II website.
Partial Cross-sections

Many reactions are possible for a given target/projectile combination.

Example.
List some of the possible reactions for a proton incident on a C-13 nucleus.

The total cross-section

\[ \sigma_T = \sum \sigma_i \]
Energetics

Laboratory reference frame

It is common in an experimental set-up that the target nucleus is at rest. The reference frame in which this is the case is therefore called the laboratory frame. We will express all of our equations in the lab frame. (This is not always the case in nuclear kinematics—reaction kinematics are often expressed in the center-of-mass or center-of-momentum frames.)
The Q-value

The Q value of the reaction is the difference between the initial and final rest energies.

\[ Q > 0 \Rightarrow \text{exothermic} \]
\[ Q < 0 \Rightarrow \text{endothermic} \]

An expression for Q is obtained by applying conservation of total relativistic energy to the reaction X(a, b)Y. (Note: we will use atomic masses.)

\[ E_i = E_f \]
The Q equation

We will derive an expression for Q that relies only on easily measured experimental parameters: the atomic masses of the reactants, the kinetic energies of the incident projectile and light reaction product, the atomic mass of the heavy product, and the scattering angle of the light product.

\[
Q = \left(1 + \frac{M_b}{M_Y}\right) T_b - \left(1 - \frac{M_a}{M_Y}\right) T_a - 2 \cos \theta_b \sqrt{\frac{M_a}{M_Y} T_a \frac{M_b}{M_Y} T_b}
\]
Reaction Kinematics and Conservation of Momentum

$X(a, b)Y$

Initial

Conservation of momentum

$$\sum p_{i,x} = \sum p_{f,x}$$

$$\sum p_{i,y} = \sum p_{f,y}$$
The Q equation

Eliminating the velocity (magnitude and direction) of the heavy product nucleus Y from the equations, we obtain the Q equation:

\[ Q = \left(1 + \frac{M_b}{M_Y}\right)T_b - \left(1 - \frac{M_a}{M_Y}\right)T_a - 2 \cos \theta_b \sqrt{\frac{M_a}{M_Y} T_a \frac{M_b}{M_Y} T_b} \]