We have seen in the previous lecture that conductors are materials which have a finite resistance. This means that when there is a potential difference across a conductor, a current results. We shall now investigate the relationship between the applied potential difference $\Delta V$ and the resulting current $I$ for conductors.
Resistors and Linear Behavior

Measurements performed in the laboratory show that some materials display a linear relationship in the behavior of their $\Delta V$ vs. $I$ plots, while others show a nonlinear behavior. Objects whose behavior is linear are called resistors; the slope of their $\Delta V$ vs. $I$ plot is their resistance, denoted $R$. The equation for the linear behavior of the potential difference vs. current plot for a resistor can therefore be written in the form

$$\Delta V_R = IR \quad \text{Ohm's Law} \quad (13.1)$$

where we have used the symbol $\Delta V_R$ to emphasize the fact that the potential difference is across the resistor (or conductor) only. This equation is called Ohm's Law.

In this lecture we will consider circuits containing combinations of resistors. We will see how to find a single resistance which is equivalent to a combination of resistors, as well as how to find the current in the circuit containing a battery or other power source and a combination of resistors.

When drawing circuits, we will use the following symbol to represent a resistor:

```
R
```

---
Series Combinations

Any type of circuit elements can be connected in series. A series combination simply means that current flowing into the first element has no choice but to flow directly into the second element. Figure 13.1 below shows examples of various generic circuit elements connected to each other in series. (The straight lines in the diagrams just represent wires through which current may flow.)

![Series Diagrams]

**Figure 13.1**

*Circuit elements in series have the same current flowing through them.*

The points A and B in Fig. 13.1 represent connecting points to the remainder of the circuit, which is not shown. (You can’t have a current flow without a closed loop for the current to flow around!)

Again, it is very important to keep in mind that, in a series combination, the current flowing out of one circuit element can only flow directly into the next one – there is no break in the wire between the elements that would give the current a choice of which direction to flow. For example, in Fig. 13.2 below, the two circuit elements shown are not connected in series.
Figure 13.2

The point in Fig. 13.2 between elements 1 and 2 where the wire splits and the current has a choice of where to go is called a node. Two circuit elements in series can never have a node between them.

Power Sources in Series

As an example of specific circuit elements in series, let’s consider first the case in which two batteries or power sources are connected in series. We will denote these two sources as $\Delta V_1$ and $\Delta V_2$. (Generically, we will denote the voltage of a single power source in a circuit by $\Delta V_s$. However, since there are two power sources in the circuit segment of Fig. 13.3, we use the subscripts 1 and 2.) It is important to remember that the long terminal is at a higher potential than the short terminal in the circuit symbol for a battery or power source by an amount $\Delta V$.

Figure 13.3

Starting at point A in Fig. 13.3 (a) and moving upwards, we increase potential (minus to plus) by an amount $\Delta V_1$, as we move from point A to point B:
\[ V_B = V_A + \Delta V_1. \quad (13.2) \]

Likewise, as we go from B to C, we’re again going from minus to plus, so our potential increases by \( \Delta V_2 \), so that

\[ V_C = V_B + \Delta V_2. \quad (13.3) \]

The net change in potential between points A and C is thus

\[ V_C = V_B + \Delta V_2 = V_A + \Delta V_1 + \Delta V_2 \quad (13.4) \]

or

\[ \Delta V_{AC} = V_C - V_A = \Delta V_1 + \Delta V_2. \quad (13.5) \]

Therefore, if \( \Delta V_1 = 5.0 \text{ V} \) and \( \Delta V_2 = 10 \text{ V} \), for example, we would measure a voltage of 15 V if we placed the black lead of a voltmeter at point A and the red lead at point C.

On the other hand, in Fig. 13.3 (b), we get that

\[ V_B = V_A + \Delta V_1 \quad \text{and} \quad V_C = V_B - \Delta V_2, \quad (13.6) \]

since, in going from point B to point C we’re moving from plus to minus, which is a negative change in potential. Thus,

\[ \Delta V_{AC} = V_C - V_A = \Delta V_1 - \Delta V_2. \quad (13.7) \]

If \( \Delta V_1 = 5.0 \text{ V} \) and \( \Delta V_2 = 10 \text{ V} \), our voltmeter would read a voltage of \(-5 \text{ V}\) if the black lead is at point A and the red lead at point C.

**Resistors in Series**

If the two elements being connected in series are *resistors*, then the single resistance that acts the same as the series combination of both resistors, often called the *equivalent resistance*, \( R_e \), is simply given by

\[ R_e = R_1 + R_2. \quad (13.8) \]

The circuit symbol for a resistor is a *zig-zag* line. Thus, using circuit symbols, we can express the replacement of a pair of resistors in series with the equivalent resistance as follows:
Figure 13.4

We can therefore simplify a circuit diagram by taking two resistors combined in series and replacing them by the single equivalent resistance $R_e$.

It is also possible to have more than two resistors connected in series. When this is the case, the same rule for finding the equivalent resistance $R_e$ applies. For example, for *three* resistors combined in series, the equivalent resistance is given by

$$R_e = R_1 + R_2 + R_3.$$  \hspace{1cm} (13.9)
Parallel Combinations

Circuit elements that are in *parallel* have two characteristics: 1) they have the same potential difference across them, and 2) they’re connected between the same two nodes in a circuit.

Consider Fig. 13.5 which shows two circuits which look different, but which are actually electrically equivalent.

![Parallel Circuits Diagram](image)

**Figure 13.5**

You should have seen from your measurements in lab that the voltage does not change along a length of wire, regardless of whether it’s bent or split (or at least the change is very small). This is because the resistance of the wire is very small. From Ohm’s law, we must have that

\[
\Delta V_{wire} = IR_{wire} , \quad \text{(13.10)}
\]

where \( I \) is the current flowing through the wire. It then follows that, if \( R_{wire} \) is very close to zero, then \( \Delta V_{wire} \) will also be very close to zero. When solving problems (unless told otherwise), we will always assume that \( R_{wire} = 0 \), so that \( \Delta V_{wire} = 0 \). (This is usually a very good approximation, as we saw in lab.) It therefore follows that, whatever the voltage is at point A in Fig. 13.5, the voltage must be the same at points B, C, and D.
(since we’ve only moved along wire to go to any of these points).

Likewise, let’s say that the voltage at point $H$ has some value that we shall denote $V_H$. Then the voltage at points G, E, and F must also be $V_H$. This means that the voltage difference across circuit element 1 and circuit element 2 must be the same:

$$\Delta V_1 = \Delta V_2 = V_A - V_H,$$  \hspace{1cm} (13.11)

the voltage between the two ends of the elements. This demonstrates the first characteristic of circuit elements in parallel:

---

**Circuit elements in parallel have the same voltage across them.**

---

The second characteristic of circuit elements connected in parallel is very straightforward to see from Figs. 13.5. Recall from Figs. 13.2 that a node is a place where a wire splits and current has a *choice* of where to go. Examination of Figs. 13.5 shows us that there are two nodes in the portion of circuit shown: one node at point B and the other at point G. The two ends of both circuit elements in parallel are connected at these two nodes. (You can go from point C to point D or from point E to point F by following just wire.) This demonstrates the second characteristic of circuit elements in parallel:

---

**Circuit elements in parallel are connected between the same two nodes in a circuit.**

---

**Power Sources in Parallel**

It is not uncomon to connect a number of batteries which have the same voltage in parallel with one another. Remember that circuit elements connected in parallel are connected between the same two nodes in a circuit. This means that all of the batteries serve to keep the voltage difference between the two common nodes at the same value. So what purpose would this serve? Simply that batteries represent a finite energy supply (relative to other power sources which can have an external energy source, such as the wall outlet in your lab). If you need 10 $V$ in a circuit and you hook up one 10-$V$ battery, then depending on how much current is demanded by the circuit, the battery may “go dead” after a relatively short period of time. Increasing the number of 10-$V$ batteries connected in parallel will serve to keep the potential difference at 10 $V$, as before, but now the circuit will “run” for a longer period of time since there is a greater energy reservoir in the combination of batteries. It is otherwise very bad practice to connect two power sources in parallel (they just end up fighting one another), so we will not address this type of connection further.
Resistors in Parallel

For two resistors connected in parallel, the equivalent resistance can be obtained by solving for $R_e$ in the following equation:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2}.$$  \hspace{1cm} (13.12)

![Diagram of resistors in parallel](image)

**Figure 13.6**

The same rule above still applies if *more* than two resistors are connected in parallel. For example, the rule for three resistors is:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$  \hspace{1cm} (13.13)
Example 13.1

Find the equivalent resistance for each of the following combinations of resistors.

(a)

\[ \begin{align*}
A & \quad 10\Omega \\
& \quad \downarrow \\
& \quad 22\Omega \\
& \quad \downarrow \\
& \quad B
\end{align*} \]

(b)

\[ \begin{align*}
A & \quad 1k\Omega \\
& \quad \downarrow \\
& \quad 3k\Omega \\
& \quad \downarrow \\
B & \quad \uparrow
\end{align*} \]

(c)

\[ \begin{align*}
A & \quad 2k\Omega \\
& \quad \downarrow \\
& \quad 4k\Omega \\
& \quad \downarrow \\
B & \quad \uparrow
\end{align*} \]
Answers: (a) 32 \Omega \quad (b) 4 \text{k}\Omega \quad (c) 1.3 \text{k}\Omega \quad (d) 2.9 \text{k}\Omega
Solution to Example 13.1

Find the equivalent resistance for each of the following combinations of resistors.

(a) Any current entering point A has no choice but to move through the first resistor and then directly into the second. These two resistors are therefore in series, so the equivalent resistance is equal to

\[ R_e = R_1 + R_2 = 32 \Omega. \]

(b) Any current entering point A has no choice but to move through the first resistor and then directly into the second toward point B. These two resistors are therefore in series, so the equivalent resistance is equal to

\[ R_e = R_1 + R_2 = 4 \, k\Omega. \]

(c) Current entering point A moves down the wire and reaches a node, or junction point, where it has a choice of which direction to go. This means that the two resistors cannot be in series. This does not necessarily mean that they are in parallel, though—we must check the conditions for a parallel connection to see if they are met. First, the resistors must be connected between the same two junction points. As we move down from point A we reach the first junction point where current has a choice of which direction to go. If we start at point B and move upwards, we will reach the second junction point. One end of each of the two resistors is connected to the first junction point, and the other end of each resistor is connected to the other junction point. Thus the two resistors are connected between the same two junction points. It also must be true that the potential difference across each resistor must be the same. The potential difference would be different if, say, there were another resistor between the ends of the two resistors attached to the top node. This is not the case, so the two potential differences across the two resistors are the same, and the two resistors are indeed connected in parallel. The equivalent
resistance is therefore given by

\[
\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2\,\text{k}\Omega} + \frac{1}{4\,\text{k}\Omega} = \frac{3}{4\,\text{k}\Omega},
\]

or \( R_e = 1.3 \,\text{k}\Omega. \)

Seeing that current coming into point A immediately bumps into a node (junction point) and has a choice of which direction to flow means that the resistors are not in series. Let's check for parallel. Let \( P_j \) (node 1) stand for the junction point that we encounter just below point A, and let \( P_2 \) be the node just to the left of point B. From the tops of the 5 k\Ω and the 10 k\Ω resistors and the left of the 20 k\Ω resistor, we can move directly to point \( P_j \) following only wire (just lines in the circuit diagram) without crossing over any other circuit elements (other resistors in this case). Thus one end of these resistors is connected directly to junction point \( P_j \). Also, the other ends of these resistors are connected directly to junction point \( P_2 \). This means that all three resistors are connected between the same two junction points. Since there are no other circuit elements between the ends of these resistors and the nodes \( P_j \) and \( P_2 \), it follows that the potential differences across each of these three resistors must be the same. These three resistors are therefore in parallel, so the equivalent resistance can be computed from

\[
\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5\,\text{k}\Omega} + \frac{1}{10\,\text{k}\Omega} + \frac{1}{20\,\text{k}\Omega} = \frac{7}{20\,\text{k}\Omega},
\]

or \( R_e = 2.9 \,\text{k}\Ω. \)
Mixed Combinations

Often a combination of circuit elements does not involve purely series or parallel combinations, but rather a mixture of the two. In such cases, it is always safest to simply find two resistors which are either in series or parallel, find the corresponding equivalent resistance for those two, and redraw the circuit with a single equivalent resistance drawn to replace the two that were combined. Then repeat this procedure until there is just one equivalent resistance left.

A note about notation:

When a total equivalent resistance is found for a mixed combination of more than two resistors, we will tend to call the total equivalent resistance \( R_e \). What do we then call the intermediate equivalent resistances? We use the labels of the resistances being combined to label the corresponding equivalent resistance. For example, if we combined resistors \( R_2 \) and \( R_3 \) in parallel, we will call the corresponding equivalent resistance \( R_{23} \). If we then combine \( R_{23} \) in series with \( R_1 \), we will call the result \( R_{231} \). If this equivalent resistance is then combined with resistor \( R_4 \) in parallel, and there are no other resistors left to combine, then we will write \( R_e = R_{2314} \).

The next example demonstrates this notation.
Solution to Example 13.2

Find the equivalent resistance of the portion of circuit shown below between terminals A and B, where \( R_1 = 2.0 \, \text{k}\Omega \), \( R_2 = R_3 = 5.0 \, \text{k}\Omega \), and \( R_4 = 10 \, \text{k}\Omega \).

We start off by examining the resistors and looking for a pair of resistors that are either in series or parallel. It is tempting to say that resistors \( R_2 \) and \( R_3 \) are in parallel (since they look like they are in parallel!), but they are not. The top two ends of these resistors are connected directly to the same node (just above \( R_2 \)), but the bottoms two ends are not, since the resistor \( R_4 \) gets in the way, making the potential difference across resistors \( R_2 \) and \( R_4 \) different from one another. Combining \( R_2 \) and \( R_4 \) in parallel is therefore \textit{not} an option. However, a quick inspection should show you that resistors \( R_3 \) and \( R_4 \) are in series, since any current flowing down into \( R_3 \) has \textit{no choice} but to flow into resistor \( R_4 \). We thus calculate the equivalent resistance for this combination (called \( R_{34} \)), and then redraw the circuit showing just \textit{one} resistor instead two.

**Series: \( R_3 \) and \( R_4 \) (same current)**

\[
R_{34} = R_3 + R_4 = 15 \, \text{k}\Omega.
\]
Inspection of this redrawn circuit should show that the two resistors $R_2$ and $R_{34}$ are in parallel (since the two tops and bottoms of the resistors are directly connected to the same node without any other resistors getting in the way). Thus:

**Parallel: $R_2$ and $R_{34}$ (same voltage)**

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}}$$

or

$$R_{234} = 3.75 \text{ k}\Omega.$$

![Diagram of parallel resistors]

We finally have only two resistors remaining that are (hopefully!) clearly in a series combination:

**Series: $R_1$ and $R_{234}$ (same current)**

$$R_e = R_{1234} = R_1 + R_{234} = 5.75 \text{ k}\Omega.$$

![Diagram of series resistors]

We have reduced the given circuit to a single resistor, which is the equivalent resistance for the circuit. This procedure will be extremely important in the next example, and especially in the next lecture when we start solving real circuits with power sources in
them (so we actually have current flow!). Note in the solution above that we listed how each resistor was being combined, which resistors were being combined, and the consequence of them being combined (same current or voltage) when working through the combinations of resistors. This will be very helpful when we get to the more difficult problems in the next lecture—do yourself a favor and get into the habit of writing out all of this information and redrawing the circuit each time you combine two (or three) resistors in series or parallel.
Ohm's Law

The circuit diagrams that we've been drawing so far have only been portions of circuits. If these were actual circuits, then no current would flow for two reasons: 1) there have been no power sources in most of the circuits drawn, and 2) current will not flow without having a complete loop to flow around.

We now consider the simplest circuit containing a battery and resistor in a closed loop—the series resistive circuit shown in Fig. 13.7. (Four points A, B, C, and D are labeled in the circuit diagram for easy reference.)

Point A is connected to ground (the symbol at the bottom-left of the circuit diagram), meaning that $V_A = 0$. Since the circuit contains a closed loop and a power source ($\Delta V_s$—the "s" subscript refers to power source) there will, in general, be current flow. The power source acts to take positive charges with no energy (since $V_A = 0$) from its negative terminal (at the lower end of the circuit symbol for $\Delta V_s$ in Fig. 13.7), gives them energy equal to $Q\Delta V$, and then pushes them out of the positive terminal (at the top of the circuit symbol for $\Delta V_s$ in Fig. 13.7). This charge then loses energy as it moves around the circuit until it reaches point A (actually by the time it reaches point D!) where it's back to ground and its energy is zero. This results in a current flow I as shown in the circuit of Fig. 13.7: out the top of the battery, around the loop, and back into the bottom of the battery. Since everything is an series, each circuit element must see exactly the same current at each instant of time!

One thing that we know for sure is that the electrostatic potential (or voltage) of a charge at point A is zero, $V_A = 0$, no matter how we get there. The potential at a given point cannot have two different values at one time! This simple idea is all we need to determine the current in the circuit.

We first note that, as we move from point A to point B, we are crossing the power source from the negative terminal to the positive terminal—this means a positive change in potential, $+\Delta V_s$. Furthermore, since current flows towards lower potential (just as water flows towards lower gravitational potential, meaning a smaller height), it follows
that the potential as we cross the resistor from point C to point D decreases by an amount we will call $\Delta V_R$, so the change in potential as we move from C to D is $-\Delta V_R$.

What is $\Delta V_R$? We know from Ohm's Law that $\Delta V_R = IR$. As already noted, as we move along only wire from point B to point C, or from point D to point A, the change in potential is 0.

Now, let's move around the circuit in Fig. 13.7 (reproduced for convenience), considering the changes in potential each step of the way. Starting at point A, and going around clockwise, we have that $V_B = V_A + \Delta V_s$, $V_C = V_B$, $V_D = V_C - \Delta V_R$, and $V_A = V_D$.

(Carefully read through the previous equations while examining Fig. 13.7 to make sure that you understand where they are coming from!) Putting this all together (starting with the last equation and substituting in as we work backwards to the first equation) gives

$$V_A = V_A + \Delta V_s - \Delta V_R,$$  \hspace{1cm} (13.14)

or

$$V_A = V_A + \Delta V_s - IR.$$  \hspace{1cm} (13.15)

If we look at Eq. (13.15), it just says that, starting at point A ($V_A$), if we go around the loop clockwise, we can just add up the changes in potential ($+\Delta V_s - IR$) until we get back to point A, at which point the potential must once again be $V_A$. This then tells us that

$$0 = \Delta V_s - IR,$$  \hspace{1cm} (13.16)

or

$$I = \frac{\Delta V_s}{R}.$$  \hspace{1cm} (13.17)

Note that, if $V$ is in volts ($V$) and $R$ is in ohms ($\Omega$), then $I$ will be in amps ($A$). Therefore, if $\Delta V_s = 15$ V and $R = 2.0$ k$\Omega = 2,000$ $\Omega$, then the current in the circuit is equal to $I = 0.0075$ A = 7.5 mA, for example.
Ohm Again!

Just for demonstration purposes, let’s work out the current in the previous circuit again, but this time let’s start at point B (it shouldn’t matter where we start) and go around counter-clockwise (it also should not matter in which direction we go).

![Circuit Diagram]

As we cross the power source from point B to point A, we go from positive to negative, so the change in potential is \(-\Delta V_s\). Also, the change in potential along the wire from A to D is zero, as discussed previously. Then, when we reach point D and cross the resistor toward point C. We are moving against the current flow (since the power source pushes current out of its positive terminal and the charges flow around and back into the negative terminal). Again, since current flows towards lower potential, heading against the current flow means that we must be moving toward a higher potential, so we have a positive change in potential: \(+\Delta V_R = +IR\). Thus, starting at point B and adding up the changes in potential counter-clockwise, we get that

\[
V_B - \Delta V_s + IR = V_B ,
\]

(13.18)

or

\[
I = \frac{\Delta V_s}{R},
\]

(13.19)

as before.

When solving a circuit problem, we will not usually label all of the various points and break things down as much as we did in the previous discussion and in Fig. 13.7. However, if you find that you’re having troubles, this is always a valid way to proceed.

The next example shows how to find the current provided to the circuit by a power source when a combination of resistors is present. This example will be very useful in the next lecture when we go into the calculation of currents in a bit more detail.
Solution to Example 13.3

Find the current provided by the power source in the following circuit. Data: $\Delta V_s = 9.0$ V, $R_1 = 1.0 \, \text{k}\Omega$, $R_2 = 2.0 \, \text{k}\Omega$, $R_3 = 5.0 \, \text{k}\Omega$.

![Circuit Diagram]

In order to find the current supplied by a power source when there is more than one resistor present in the circuit we first combine all of the resistors until there is only one (the equivalent) resistance left. This is almost equivalent to the solution in the previous example, and will be covered rather quickly in this example. (Look back through the last example if you still feel shaky about doing this!)

**Parallel: $R_2$ and $R_3$ (same voltage: $\Delta V_2 = \Delta V_3 = \Delta V_{23}$)**

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3},$$

or

$$R_{23} = 1.43 \, \text{k}\Omega.$$
Series: $R_1$ and $R_{23}$ (same current: $I_1 = I_{23} = I_{123}$)

$$R_c = R_{123} = R_1 + R_{23} = 2.43 \text{ k}\Omega.$$ 

We now have exactly the situation described in the previous section. Starting at point $A$ and moving clockwise around the loop, we get that

$$\Delta V_s - I_s R_c = 0$$

or

$$I_s = \Delta V_s / R_c = 3.71 \times 10^{-3} \text{ A} = 3.7 \text{ mA}.$$
The Voltage Divider

To finish up this lecture we examine a straightforward but very important circuit. This simple circuit is so common that it is given a special name—the voltage divider.

The voltage-divider circuit is shown below in Fig. 13.8.

![The Voltage Divider](image)

**Figure 13.8**

The voltage-divider circuit shown above contains two resistors, $R_1$ and $R_2$, and a power source having a constant voltage that we shall denote $\Delta V_{\text{in}}$ (shown as “Vin” in the circuit diagram above), which we will refer to as the **input voltage**. The output voltage $\Delta V_{\text{out}}$ (“Vout” in the circuit diagram) is taken to be between the two points labeled $B$ and $C$ in Fig. 13.8—that is, across the resistor $R_2$.

*The circuit diagram shows two leads connected to the circuit at points $B$ and $C$. It is to be understood that these leads are connected to some sort of voltmeter or other measuring device that can read the output voltage in the circuit. It is further understood that no current is allowed to flow out of or into the voltmeter or measuring device along the leads from points $B$ and $C$ toward the terminals labeled “Vout” in the circuit diagram above. (Voltmeters have a very high resistance and thus effectively allow no current to flow through them.) Since no current flows through the leads connected to points $B$ and $C$ we can ignore (or erase!) them when computing the current flow through the circuit.*

It should be clear that the resistors $R_1$ and $R_2$ are in a **series** connection. The series combination of these two resistors results in the single equivalent resistance $R_{12} = R_1 + R_2$. When the two resistors $R_1$ and $R_2$ in Fig. 13.8 are replaced with the single equivalent resistance $R_{12}$ we find that we end up with a circuit that is equivalent to the circuit shown in Fig. 13.7 in the section on Ohm’s Law. The current in our circuit is then solved for in exactly the same way as the current $I$ was solved for in the circuit of Fig. 13.7. This current, as given by Eq. (13.17), is then equal to
\[ I = \frac{\Delta V_{in}}{R_{12}} = \frac{\Delta V_{in}}{R_1 + R_2}, \quad (13.20) \]

where we have used \( \Delta V_s = \Delta V_{in} \) and \( R = R_{12} = R_1 + R_2 \) for our circuit. Since \( R_{12} \) comes from the series combination of \( R_1 \) and \( R_2 \), it follows that the current \( I \) flowing out of the voltage source must also flow through each of the resistors \( R_1 \) and \( R_2 \). From Ohm’s law, it then immediately follows that the output voltage, taken to be the voltage across resistor \( R_2 \) in Fig. 13.8, is given by

\[ \Delta V_{out} = IR_2 = \left( \frac{\Delta V_{in}}{R_1 + R_2} \right) R_2. \quad (13.21) \]

One final step in algebra then shows us that the output voltage is related to the input voltage by the equation

\[ \Delta V_{out} = \left( \frac{R_2}{R_1 + R_2} \right) \Delta V_{in}. \quad (13.22) \]

The input voltage for the circuit, \( \Delta V_{in} \), is divided into two parts—one part across resistor \( R_1 \) and the other part across resistor \( R_2 \). With the output taken to be across resistor \( R_2 \), Eq. (13.22) shows us that the output voltage is just a fraction of the input voltage, that fraction being completely determined by the values of the resistances \( R_1 \) and \( R_2 \).

For example, if \( R_1 = 10 \text{ k}\Omega \) and \( R_2 = 20 \text{ k}\Omega \), then \( \Delta V_{out} = (2/3) \Delta V_{in} = (0.67) \Delta V_{in} \), while if \( R_1 = 2.0 \text{ k}\Omega \) and \( R_2 = 680 \Omega \), then \( \Delta V_{out} = (0.25) \Delta V_{in} \). *(Verify these values!)*

We finish up this section with two final notes about the voltage-divider circuit. The first note involves a *variable resistor* known as a *potentiometer* (or a *rheostat* in the case of high-power circuits).

A **potentiometer** is a single resistor \( R \) that is split into two parts, \( R_1 \) and \( R_2 \), such that \( R_1 + R_2 \) is always equal to the total resistance \( R \): \( R_1 + R_2 = R \). The potentiometer usually has a knob that can be turned. As this knob is turned the resistance \( R_1 \) gets smaller and the resistance \( R_2 = R - R_1 \) gets larger. The way this works is that, as the knob is turned, a contact point inside the potentiometer that defines the breaking-off point between \( R_1 \) and \( R_2 \) is moved along a resistive wire or strip that defines the total resistance \( R \). This can be seen in the way the circuit symbol for a potentiometer is drawn. As shown in Fig. 13.9 below, the circuit element for the potentiometer is just the circuit symbol for a resistor with a contact point with an arrow drawn in towards the middle of the resistor. The portion of the resistor above the contact point is labeled as resistor \( R_1 \) in the figure, and that below the contact point is labeled as \( R_2 \). If the resistances \( R_1 \) and \( R_2 \) from a potentiometer are used in the voltage-divider circuit as
shown in Fig. 13.9, then the output voltage can be continuously varied from zero to some maximum value as the potentiometer knob is turned and the values of $R_1$ and $R_2$ are changed. Such a voltage-divider set-up is commonly used as a volume control in radios or stereo systems, or as fine-adjustment knobs on oscilloscopes or other electronic instruments. (See HW13 #10.)

![Volt Divider Circuit Diagram](Image)

**Figure 13.9**

The second comment is that we will encounter the voltage-divider circuit again in our study of electronics. Indeed, it will play an important role in our discussion of high- and low-pass filter circuits when we study alternating-current (ac) circuits. In that discussion we will define the magnitude of the ratio of the output voltage to the input voltage in Eq. (13.22), $|\Delta V_{\text{out}}/\Delta V_{\text{in}}|$, to be the gain of the circuit, denoted $G$. (See HW13 #9.)
Sample Quiz 13

1. Ohm’s law is a relationship between the _______ and _______ of a resistor.
   a. current and charge. b. voltage and time. c. current and voltage. d. voltage and charge. e. voltage and potential difference.

2. A characteristic of two circuit elements in series is that they have the same
   a. current. b. voltage. c. node. d. time. e. power supply.

3. A characteristic of two circuit elements in parallel is that they have the same
   a. current. b. voltage. c. node. d. time. e. power supply.

4. The equivalent resistance is the resistance that
   a. is equal to the sum of two resistances. b. acts like only one of the resistors is present. c. acts like the resistances divided into one another. d. acts the same as the original resistances. e. equals the sum of the resistances divided by the product of the resistances.

5. If you move along a resistor in a direction what is against the direction of current flow through that resistor then
   a. the voltage increases. b. the current is negative. c. the voltage decreases. d. the power supply is negative. e. the voltage must remain the same (Ohm’s law).

Answers
Answers to Sample Quiz 13

1. c
2. a
3. b
4. d
5. a
1. Find the equivalent resistance of the following section of circuit between points A and B:

![Diagram of a circuit with resistances 10 Ω and 30 Ω connected in series between points A and B.]

2. Find the equivalent resistance of the following section of circuit between points C and D:

![Diagram of a circuit with resistances 500 Ω, 700 Ω, and 1 kΩ connected in series between points C and D.]

3. Find the equivalent resistance of the following section of circuit between points P and Q:

![Diagram of a circuit with resistances 15 Ω and 20 Ω connected in series between points P and Q.]

4. Find the equivalent resistance of the following section of circuit between points E and F:
5. Find the equivalent resistance of the following section of circuit between points A and B:

6. Find the current supplied to the circuit by the power source.

7. Find the current supplied to the circuit by the power source.

8. Find the current supplied to the circuit by the power source.
9. Consider the voltage-divider circuit of Fig. 13.8 in which $\Delta V_{in} = 50 \text{ V}$, $R_1 = 100 \text{ k}\Omega$, and $R_2 = 1.1 \text{ k}\Omega$. (a) What is the current in the circuit? (Work this out from scratch—do not just use Eq. 13.20!) (b) What is the output voltage of the circuit? (c) The gain of the circuit, $G$, is defined to be $G = \Delta V_{out} / \Delta V_{in}$ (that is, how much we get out compared to how much we put in—much like the miles-per-gallon for a car...). What is the value of the gain for this circuit?

10. A potentiometer used in a voltage-divider circuit (Fig. 13.8) has a total resistance of $1.5 \text{ k}\Omega$. As the potentiometer knob is turned from $0^\circ$ to $360^\circ$ the resistance $R_1$ in the circuit varies linearly from $1.5 \text{ k}\Omega$ to $0 \text{ k}\Omega$. The input voltage for the circuit is $45 \text{ V}$. (a) What is the value of the resistance $R_2$ in the voltage-divider circuit of Fig. 13.8 when the potentiometer knob is at $0^\circ$? (b) What is the value of the resistance $R_2$ in the voltage-divider circuit when the potentiometer knob is at $120^\circ$? (c) What is the output voltage from the voltage-divider circuit when the knob is at $0^\circ$? (d) What is the output voltage from the voltage-divider circuit when the knob is at $120^\circ$? (e) What is the output voltage from the voltage-divider circuit when the knob is at $360^\circ$?

11. Let's say that we have an electronic thermometer that measures the temperature of a chemical solution and outputs the temperature signal in the form of a voltage (there is a linear relationship between the voltage output and the solution temperature). We wish to connect the thermometer output to a PC that will record the temperature (voltage) variations of the solution as a function of time. The problem is that the output of the electronic thermometer can go up to $1.5 \text{ V}$, while the connector to the PC should not receive a voltage signal that exceeds $10 \text{ mV}$. To solve this problem you decide to design and construct a voltage-divider circuit. You realize that if you feed the output of the thermometer into the voltage-divider circuit, and then connect the output of the voltage divider into the PC everything should work just fine if you use the proper resistors in the voltage-divider circuit. Searching through your lab, you find that you only have resistors with resistance values $10 \text{ k}\Omega$, $68 \text{ k}\Omega$, $100 \text{ k}\Omega$, $720 \text{ k}\Omega$, $1.0 \text{ k}\Omega$, and $1.5 \text{ k}\Omega$. Which values of resistance should be used for resistors $R_1$ and $R_2$ in the voltage-divider circuit of Fig. 13.8 so that the PC will be able to read the electronic thermometer output signal?
Answers to Homework 13

1. 40 Ω
2. 2.2 kΩ
3. 8.6 Ω
4. 4.3 kΩ
5. 22.7 kΩ
6. 1.25 mA
7. 1.18 mA
8. 4.29 mA
9. (a) 42 mA  (b) 46 V  (c) 0.92
10. (a) 0 Ω  (b) 500 Ω  (c) 0 V  (d) 15 V  (e) 45 V
11. R₁ = 1.5 kΩ; R₂ = 10 Ω