Test 2

Name: __________ KEY __________ | __________ First | __________ Last

Do not open this exam until you are told to do so.

This exam consists of two parts. Part I contains 5 multiple-choice questions worth 2 points each: choose the best answer and put your answers in the boxes provided at the end of Part I. No partial credit. Part II contains 3 problems worth a total of 90 points. All work leading to a correct answer with proper units must be shown in order to receive full credit. Keep numbers out of your equations as long as possible to help clarify your reasoning. Please box-in your answers.

Ask if you don’t understand the statement of a given problem or what is being asked.

INSTRUCTIONS

• ALL WORK MUST BE DONE ON THESE PAGES. NO EXTRA SCRATCH PAPER IS ALLOWED.
• ALL NUMERICAL ANSWERS MUST HAVE APPROPRIATE UNITS.
• YOU WILL BE GRADED ON HOW WELL YOU COMMUNICATE YOUR METHOD OF SOLUTION USING SYMBOLS.
• BOX-IN YOUR ANSWERS!

Cheating of any kind will not be tolerated.

You have 1 hour to complete this exam.
Budget your time accordingly.
Part I: Multiple Choice. Choose the best answer. No partial credit. To receive credit, put your answers in the boxes at the end of this section. 2 points each.

1. Nickel (58.69 g/mol) has an interatomic spacing of \(2.22 \times 10^{-10}\) m. A 2.14-m length of nickel wire has a circular cross section of diameter 1.58 mm. Within our model of a solid, we view this wire as a number \(N_p\) of effective springs connected in parallel, and a number \(N_s\) of springs connected in series. What is the value of \(N_p\) for the nickel wire?
   
   \[ r = 7.90 \times 10^{-4} \text{ m} \]
   \[ A = \pi r^2 = 1.96 \times 10^{-6} \text{ m}^2 \]
   \[ A_{	ext{atom}} = a^2 = 4.93 \times 10^{-20} \text{ m}^2 \]
   
   \[ N_p = \frac{A}{A_{	ext{atom}}} = 3.98 \times 10^{13} \]
   
   a. \(2.79 \times 10^{12}\)  
   b. \(8.27 \times 10^{17}\)  
   c. \(1.04 \times 10^{9}\)  
   d. \(3.85 \times 10^{21}\)  
   e. \(3.98 \times 10^{13}\)

2. What is the value of \(N_s\) for the nickel wire above?
   
   a. \(2.55 \times 10^{15}\)  
   b. \(7.25 \times 10^{12}\)  
   c. \(5.47 \times 10^{13}\)  
   d. \(9.64 \times 10^{9}\)  
   e. \(4.74 \times 10^{10}\)

   \[ N_s = \frac{L}{d} = 9.64 \times 10^{9} \]

3. A car drives down a long, straight road without slipping. Which of the following statements is true about the car?
   
   a. The friction between the car and the road is static friction since there is no sliding motion between the tires and the road surface.
   b. The friction between the car and the road is static friction since the car is moving with a constant velocity.
   c. The friction between the car and the road is kinetic friction since the car is moving.
   d. The friction between the car and the road is kinetic friction since the car is moving with a constant velocity.
   e. No friction is needed since the car is moving with constant velocity.

4. Which of the following statements is true about an object moving in uniform circular motion?
   
   a. The velocity of the object is constant.  
   b. The speed of the object is constant.  
   c. The acceleration of the object is constant.  
   d. The acceleration of the object is zero.  
   e. All of these are true.

5. Which of the following statements is true about a system in static equilibrium?
   
   a. The speed of the system is zero.  
   b. The velocity of the system is zero.  
   c. The acceleration of the system is zero.  
   d. The net force acting on the system is zero.  
   e. The net torque acting on the system is zero.  
   f. All of these.

Put your multiple-choice answers here! This is the only place your answers will be graded!

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>d</td>
<td>a</td>
<td>b</td>
<td>f</td>
</tr>
</tbody>
</table>
Part II: Problems. All work must be shown using symbols, and answers must have appropriate units for full credit.

1. (30 pts) A uniform beam of mass \( M = 57 \text{ kg} \) and length \( 2.5 \text{ m} \) is attached at its left end to a wall. The right end of the beam is attached to the ceiling by means of a cord which holds the beam in a horizontal position. The cord makes an angle \( \theta = 57^\circ \) with the horizontal, as shown. A ball of mass \( m_{\text{ball}} = 14.2 \text{ kg} \) is rolled along the beam until its center-of-mass is at a distance \( x = 1.45 \text{ m} \) from the wall.

Find the values of the tension in the cord and the horizontal and vertical components of the force exerted by the wall on the beam.

\[
\begin{align*}
M &= 57 \text{ kg}, \quad L = 2.5 \text{ m}, \quad \theta = 57^\circ \\
M_{\text{ball}} &= 14.2 \text{ kg}, \quad x = 1.45 \text{ m} \\
\text{Equilibrium:} \quad \tau_{\text{net}} &= 0 = \tau_h + \tau_v + \tau_M + \tau_{\text{ball}} + \tau_T \\
0 &= +Mg \frac{L}{2} + M_{\text{ball}}g x - F_T L \sin \theta \\
\Rightarrow F_T &= \frac{Mg \frac{L}{2} + M_{\text{ball}}g x}{L \sin \theta} = 42.9 \text{ N} \\
\text{Also:} \quad F_{\text{net}_x} &= +F_T \cos \theta - F_h = 0 \\
\Rightarrow F_h &= F_T \cos \theta = 1234 \text{ N} \\
F_{\text{net}_y} &= +F_v + F_T \sin \theta - Mg - M_{\text{ball}}g = 0 \\
\Rightarrow F_v &= (M + M_{\text{ball}})g - F_T \sin \theta = 338 \text{ N}
\end{align*}
\]
2. (30 pts) A spring of spring constant 39.8 N/m is attached to the ceiling. A mass \( m = 500 \text{ g} \) is attached to the bottom of the spring and is slowly lowered until it hangs in equilibrium from the end of the spring. We will call this position of the mass the \( y = 0 \) position. The mass is then lifted by 8.5 cm and released from rest. The mass then undergoes simple-harmonic motion. You study the motion of the mass in LoggerPro and find that, at the time \( t = 0 \) in the video, the mass is 2.7 cm below the \( y = 0 \) position and moving upward with a speed of 72.0 cm/s.

\( k_s = 39.8 \text{ N/m}, \ m = 0.500 \text{ kg}, \ y_{\max} = 8.5 \text{ cm}, \ y_0 = -2.7 \text{ cm}, \ \dot{y}_0 = +72 \text{ cm/s} \)

(a) By what distance is the spring stretched from its unstretched equilibrium length when the mass \( m \) is hanging at rest at the bottom of the spring? Find \( s \).

\[
\begin{align*}
\vec{F}_p &= m\vec{g} \\
F_{\text{net}} &= m\dot{y} = 0 \\
\vec{F}_p - mg &= 0 \\
\vec{F}_p &= mg \\
k_s \cdot s &= mg \\
\Rightarrow \quad s &= \frac{mg}{k_s} = 12.3 \text{ cm}
\end{align*}
\]

(b) What is the value of the phase constant for the simple harmonic motion of the mass?

\[
\begin{align*}
y(t) &= y_{\max} \cos(\omega t + \phi) \Rightarrow y_0 = y_{\max} \cos \phi < 0 \Rightarrow \cos \phi < 0 \\
\dot{y}(t) &= \frac{dy}{dt} = -\omega y_{\max} \sin(\omega t + \phi) \Rightarrow \dot{y}_0 = -\omega y_{\max} \sin \phi < 0 \Rightarrow \sin \phi < 0
\end{align*}
\]

\[
\begin{align*}
\frac{\dot{y}_0}{y_0} &= -\frac{\omega y_{\max} \sin \phi}{y_{\max} \cos \phi} = -\frac{\omega \sin \phi}{\cos \phi} = -\tan \phi
\end{align*}
\]

Thus, \( \tan \phi = -\frac{\dot{y}_0}{y_0} = 2.99 \Rightarrow \phi = \tan^{-1}(2.99) = 1.25 \text{ rad} < \frac{\pi}{2} \)

The need the 3rd quadrant:

\[
\phi = 1.25 \text{ rad} + \pi \text{ rad} = 4.39 \text{ rad}
\]

(c) Write down the equation that describes the simple harmonic motion of the mass described above as a function of time.

\[
y(t) = [8.5 \text{ cm}] \cos [(8.92 \text{ rad/s}) t + 4.39 \text{ rad}]
\]
3. (30 pts) In an amusement-park ride called the Rotor, people are led into a room which is in the shape of a large cylinder. They stand up against the walls of the room. The room then starts rotating faster and faster, so that they feel that they are “pressed” up against the walls. At some point when the room is rotating fast enough, the floor is lowered, leaving the people “stuck” to the walls of the room. We wish to investigate the physics behind this ride.

A cylindrical room has a radius of \( R = 4.6 \, \text{m} \). A person of mass \( m = 79 \, \text{kg} \) leans against the wall of the room. The room rotates twice in a time of \( 2.58 \, \text{s} \).

(a) What is the magnitude of the “centripetal force” acting on the person while the room is rotating?

Find \( F_{\text{cent}} \).

Note: Period \( T = \frac{2.58 \, \text{s}}{2} = 1.29 \, \text{s} \)

\[
V = \frac{\text{circ}}{\text{time}} = \frac{2\pi R}{T} = 22.4 \, \frac{m}{s}
\]

Thus, \( F_{\text{cent}} = m\omega^2 = \frac{mV^2}{R} = 8,620 \, \text{N} \)

(b) What is the minimum value of the coefficient of friction between the person and the wall so that the person does not slide when the floor is lowered?

For the person not to slide we must have that \( F_{\text{fry}} = 0 = F_f - mg \)

\[ \Rightarrow F_f = mg \]

For max. static friction (minimum value of \( \mu \)),

\[ \mu F_N = mg \]

But from (a), \( F_{\text{cent}} = +F_N = 8,620 \, \text{N} \).

Thus,

\[ \mu_{\text{min}} = \frac{mg}{F_N} = 0.0898 \]