Test 1

Name:  

First  |  Last

Please check your lab section:  TR 8:00 AM (Robertson)  WF 2:20 PM (Montemayor)

Do not open this exam until you are told to do so.
This exam consists of two parts.  Part I contains 5 multiple-choice questions worth 2 points each: choose the best answer and put your answers in the boxes provided at the end of Part I.  No partial credit.  Part II contains 3 problems worth a total of 90 points.  All work leading to a correct answer with proper units must be shown in order to receive full credit.  Keep numbers out of your equations as long as possible to help clarify your reasoning. Please _box-in_ your answers.

Ask if you don’t understand the statement of a given problem or what is being asked.

INSTRUCTIONS

- All work must be done on these pages. No extra scratch paper is allowed.
- All numerical answers must have appropriate units.
- You will be graded on how well you communicate your method of solution using symbols.
- Box-in your answers!

Cheating of any kind will not be tolerated.

You have 1 hour and 25 minutes to complete this exam.  
Budget your time accordingly.
Part I: Multiple Choice. **Choose the best answer. No partial credit. To receive credit, put your answers in the boxes at the end of this section. 2 points each.**

1. A spaceship is docked at a spacestation in orbit around a planet. The spaceship pulls out of the dock and speeds away from the planet. Which of the following graphs best shows the variation in the magnitude of the gravitational force between the spaceship and the planet as a function of distance between the two?

   ![Graphs](image)

   \[ F_G \propto \frac{1}{\Delta r^2} \]

2. Which of the following graphs best represents the behavior of the tension force magnitude in a spring as the spring is stretched beyond its unstretched length?

   ![Graphs](image)

   \[ F_{sp} \propto s \]

3. The linear mass density of a non-uniform thin rod of length L is given by \( \rho_L(x) = \frac{Ax}{L^2} \), where \( A \) is a constant and \( x \) is the distance from the left-hand end of the rod, which is at the origin. What are the SI units of the constant \( A \)?

   (a) kg  (b) kg/m  (c) kg m  (d) kg/m^2  (e) kg/m^3

   \[
   \left[ \rho_L \right] = \left[ \frac{Ax}{L^2} \right] = \frac{kg}{m} \rightarrow \rho_L = \frac{dm}{dx} \\
   \left[ A \right] \cdot \frac{m}{m^2} = \frac{kg}{m} \\
   \left[ A \right] = \frac{kg}{m}.
   \]
4. A low-friction cart is released from rest at the top of an incline. Data for the motion of the cart are obtained from LoggerPro, in which the origin is set at the initial position of the cart and the positive-x axis is set in the direction of the cart’s motion. Which of the following plots best shows the behavior of the $x$ vs. $t$ graph that would be obtained?

\[ x = \frac{1}{2} a x t^2 \rightarrow x \propto t^2 \]

5. You then take the data from the cart in the previous question and construct a plot of $x$ vs. $t^2$. Which of the following plots best shows the behavior of the $x$ vs. $t^2$ graph that would be obtained?

\[ [x] = \left( \frac{1}{2} a x \right) [t^2] + c \]

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Put your multiple-choice answers here! This is the only place your answers will be graded!

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Part II: Problems. All work must be shown using symbols, and answers must have appropriate units for full credit. Don't forget to box-in your answers!

1. (30 pts) A cart of mass \( m = 135 \text{ g} \) is given a quick push down a track starting from the top of the track. The track is inclined at an angle \( \theta = 32^\circ \). The cart is pushed with an initial speed of \( v_i = 15.7 \text{ cm/s} \).

(a) Draw a good FBD in the space below for the cart during its motion after being pushed. Be sure to clearly label your choice of axis directions. Also write out \( F_{\text{net,x}} \) and \( F_{\text{net,y}} \) as usual.

\[
F_{\text{net,x}} = +mg \sin \theta \\
F_{\text{net,y}} = +F_N - mg \cos \theta
\]

(b) Let \( t = 0 \) be the time that the cart was released after its initial push. With what speed was the cart moving at \( t = 1.5 \text{ s} \)? Use the Momentum Principle.

\[
\mathbf{MP}(x) : \quad m v_{fx} = m v_{ix} + F_{\text{net,x}} \Delta t \\
\frac{m v_{fx}}{m} = v_{ix} + g \sin \theta \Delta t \\
v_{fx} = v_{ix} + g \sin \theta \Delta t \\
v_{fx} = 7.95 \text{ m/s}
\]

(c) How far did the cart move during the 1.5-s interval after it was pushed?

\[
v_{\text{avg,x}} = \frac{v_{ix} + v_{fx}}{2} = \frac{0 + 7.95}{2} \text{ m/s} \\
x_f = x_i + v_{\text{avg,x}} \Delta t \\
x_f = 0 + 4.05 \times 1.5 \text{ s} \\
x_f = 6.08 \text{ m}
\]

(d) What was the apparent weight of the cart as it moved down the incline? Use the Momentum Principle.

\[
\mathbf{MP}(y) : \quad m v_{fy}^0 = m v_{fy}^0 + F_{\text{net,y}} \Delta t \\
\Rightarrow F_{\text{net,y}} = 0 = F_N - mg \cos \theta
\]

Thus:

\[
F_N = mg \cos \theta \\
F_N = 1.12 \text{ N}
\]
2. (30 pts) A ball is kicked from a platform that is \( h = 1.7 \text{ m} \) above the ground. The ball is kicked directly toward a wall that is \( H = 5.2 \text{ m} \) high. The ball has a mass \( m = 2.6 \text{ kg} \), and is kicked with an initial speed \( v_i = 8.2 \text{ m/s} \) at an angle of \( \theta = 52^\circ \) above the horizontal. The base of the wall is a horizontal distance \( D = 7.5 \text{ m} \) from the position where the ball was kicked.

(a) What are the \( x \) - and \( y \) - components of the initial ball's velocity?

\[
\begin{align*}
V_{ix} &= +v_i \cos \theta = 5.05 \text{ m/s} \\
V_{iy} &= +v_i \sin \theta = 6.46 \text{ m/s}
\end{align*}
\]

(b) Does the ball make it over the wall? If so, by how much? If not, by how much does it fall short? Use the Momentum Principle formalism to solve this problem. Clearly label and box-in your answer.

What is \( y \) when \( x = D \)?

\[
\begin{align*}
\text{FBD:} & \quad \overrightarrow{\text{Free, } x} = 0 \\
& \quad \overrightarrow{\text{Free, } y} = -mg
\end{align*}
\]

\[
\begin{align*}
\text{MP(x): } & \quad mv_f x = mv_i x + F_{\text{net}} x \Delta t \quad \Rightarrow \quad V_{fx} = V_{ix} = \text{const.} = 5.05 \text{ m/s} \\
& \quad \Rightarrow V_{avg x} = V_{ix} = 5.05 \text{ m/s} \\
\text{PU(x): } & \quad x_f = x_i + V_{avg x} \Delta t \\
& \quad \Rightarrow \Delta t = \frac{D}{V_{avg x}} = 1.49 \text{ s}
\end{align*}
\]

\[
\begin{align*}
\text{MP(y): } & \quad mv_f y = mv_i y + F_{\text{net}} y \Delta t \\
& \quad \Rightarrow V_{avg y} = \frac{V_{iy} + V_{fy}}{2} = -0.84 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\text{PU(y): } & \quad y_f = y_i + V_{avg y} \Delta t \\
& \quad y_f = 0.45 \text{ m} < H = 5.2 \text{ m}
\end{align*}
\]

The ball does not make it over the wall. It falls short by \( 5.2 \text{ m} - 0.45 \text{ m} = 4.75 \text{ m} \).
3. (30 pts) The Andromeda galaxy is a galaxy that is in the Local Group of galaxies, of which our Milky Way galaxy is a member. The Andromeda galaxy has a mass of about \(1.4 \times 10^{42}\) kg, while our Milky Way galaxy has a very close mass of about \(2.0 \times 10^{42}\) kg. (The rest of the galaxies in the Local Group are much smaller.) Relative to some arbitrary origin and axes, the Milky Way galaxy is at the position \(\vec{r}_{\text{mw}} = (-0.93, -0.25, 9.5) \times 10^{19}\) km, while Andromeda is at the position \(\vec{r}_{\text{an}} = (0.78, -1.6, 8.5) \times 10^{19}\) km. We wish to look at the force exerted on Andromeda by the Milky Way.

(a) What is the distance \((\text{in SI units})\) from the Milky Way to the Andromeda galaxy?

\[
\Delta^2 = \vec{r}_{\text{an}} - \vec{r}_{\text{mw}} \\
\Delta^2 = \langle 1.71, -1.35, -1.00 \rangle \times 10^{19} \text{ km} \\
\quad = \langle 1.71, -1.35, -1.00 \rangle \times 10^{22} \text{ m} \quad \text{\textit{SI units}}
\]

Thus, \(\text{Distance} = |\Delta\vec{r}| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = 2.4 \times 10^{22} \text{ m}\)

(b) What is the \((\text{vector})\) force exerted on Andromeda by the Milky Way? \((\text{Give the answer in component form.})\)

\[
\vec{F}_G = G \frac{M_{\text{an}} M_{\text{mw}}}{\Delta r^2} \quad \Delta \vec{r} = \frac{\Delta \vec{r}}{|\Delta \vec{r}|} = \langle 0.71, -0.56, -0.42 \rangle
\]

\[
\vec{F}_G = G \frac{M_{\text{an}} M_{\text{mw}}}{\Delta r^2} = 3.24 \times 10^{29} \text{ N} \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2.
\]

Thus: \(\vec{F}_G = \langle -2.3 \times 10^{29} \text{ N}, 1.8 \times 10^{29} \text{ N}, 1.4 \times 10^{29} \text{ N} \rangle\)

and \(\vec{F}_G = \langle -2.3, 1.8, 1.4 \rangle \times 10^{29} \text{ N}\) \textit{Force on Andromeda}

(c) What is the position of the \textit{center-of-mass} of the Milky Way-Andromeda system? \((\text{Give the answer in SI units in component form.})\)

\[
\vec{r}_{\text{cm}} = \frac{M_{\text{mw}} \vec{r}_{\text{mw}} + M_{\text{an}} \vec{r}_{\text{an}}}{M_{\text{mw}} + M_{\text{an}}} \\
\vec{r}_{\text{cm}} = \frac{0.59 \vec{r}_{\text{mw}} + 0.41 \vec{r}_{\text{an}}}{0.59 + 0.41} \\
\vec{r}_{\text{cm}} = \langle -0.23, -0.80, 9.09 \rangle \times 10^{22} \text{ m}
\]