Mass in Horizontal Spring

\( m = 0.295 \, \text{kg}, \quad x_{\text{max}} = 0.085 \, \text{m} \),

At \( t = 0 \): \( x_0 = -0.029 \, \text{m}, \quad v_0 = +0.12 \, \text{m/s} \)

(a) Find \( T \).

\[ x(t) = x_{\text{max}} \cos(\omega t + \phi) \Rightarrow x_0 = x_{\text{max}} \cos \phi \]

Also:

\[ v_x(t) = \frac{dx}{dt} = -\omega x_{\text{max}} \sin(\omega t + \phi) \Rightarrow v_0 = -\omega x_{\text{max}} \sin \phi. \]

It then follows that:

\[ \sin \phi = -\frac{v_0}{\omega x_{\text{max}}} \quad \text{and} \quad \cos \phi = \frac{x_0}{x_{\text{max}}}. \quad (A) \]

We wish to eliminate \( \phi \) to solve for \( \omega \) (one then \( T \)).

Pythagorean theorem:

\[ \sin^2 \phi + \cos^2 \phi = 1 \]

\[ \frac{v_0^2}{\omega^2 x_{\text{max}}^2} + \frac{x_0^2}{x_{\text{max}}^2} = 1. \]

\[ v_0^2 + \omega^2 x_0^2 = \omega^2 x_{\text{max}}^2 \]

\[ \Rightarrow \omega = \frac{v_0}{\sqrt{x_{\text{max}}^2 - x_0^2}} = 1.50 \, \text{rad/s} = \frac{2\pi}{T} \]

so that \( T = \frac{2\pi}{\omega} = 4.18 \, \text{s} \)

(b) Find \( \phi \).

From (A), \( \frac{\sin \phi}{\cos \phi} = \tan \phi = -\frac{v_0}{\omega x_0} = +2.76 \)

\[ \Rightarrow \phi = \tan^{-1}(2.76) \approx 1.22 \, \text{rad} \approx 70^\circ. \quad (1^{\text{st}} \text{quadrant}) \]

But from (A), \( \sin \phi < 0 \) and \( \cos \phi < 0 \), so \( \phi \) must be in the \( 3^{\text{rd}} \) quadrant. The phase constant must therefore be

\[ \phi = 1.22 + \pi = 4.36 \, \text{rad} \]

(about 250°)
Given: \( m = 1.0 \text{ kg}, \quad L = 2.0 \text{ m} \)
\( \Theta_{\text{max}} = 15^\circ, \quad V_i = 0 \text{ at } t = 0 \).

Find the equation for the motion of the pendulum, \( \Theta(t) \), and the period of the motion.
Make the small-angle approximation.

\[ F_{\text{net}} = F_T \sin \Theta \]
\[ F_{\text{net}} = F_T \cos \Theta - mg. \]

We make the small-angle approximation.

Under this approximation, the motion is basically only in the x-direction (the mass just swings back and forth a little bit, mostly horizontally). Thus, \( \Delta y \equiv 0 \). We also have from the top figure that \( x = L \sin \Theta \). But under the small-angle approximation, \( \tan \Theta \equiv \sin \Theta \equiv \Theta \text{ (in radians)} \). Thus,
\[ x \equiv L \Theta \Rightarrow \frac{dx}{dt} = L \frac{d\Theta}{dt} \text{ and } \frac{d^2x}{dt^2} = L \frac{d^2\Theta}{dt^2}. \]

**MP (y):**
\[ F_{\text{net}} = m g \Rightarrow F_T \cos \Theta - mg = 0 \Rightarrow F_T = \frac{mg}{\cos \Theta}. \]

**MP (x):**
\[ F_{\text{net}} = m \frac{dx}{dt} \Rightarrow - F_T \sin \Theta = m L \frac{d^2\Theta}{dt^2}. \]

Substituting for \( F_T \):
\[ - \frac{mg}{\cos \Theta} \sin \Theta = mL \frac{d^2\Theta}{dt^2}. \]

\[ L \frac{d^2\Theta}{dt^2} = -g \tan \Theta. \quad \text{But } \tan \Theta \equiv \Theta, \text{ so we get: } \]
\[ \frac{d^2\Theta}{dt^2} = - \left( \frac{g}{L} \right) \Theta. \]

This is the form of the differential equation for simple harmonic motion for \( \Theta(t) \), where the angular frequency is given by \( \omega = \sqrt{\frac{g}{L}} = 2.21 \text{ rad/s} \).

But \( \omega = \frac{2\pi}{T} \), so the period of the motion is:
\[ T = \frac{2\pi}{\omega} = 2.84 \text{ s} \]