Chapter 14 Introduction to Time Series Regression and Forecasting

14.1. (a) Since the probability distribution of Y_t is the same as the probability distribution of Y_{t-1} (this is the definition of stationarity), the means (and all other moments) are the same.

(b) $E(Y_t) = \beta_0 + \beta_1 E(Y_{t-1}) + E(u_t)$, but $E(u_t) = 0$ and $E(Y_t) = E(Y_{t-1})$. Thus $E(Y_t) = \beta_0 + \beta_1 E(Y_t)$, and solving for $E(Y_t)$ yields the result.

14.2. (a) The statement is correct. The monthly percentage change in IP is $\frac{IP_t - IP_{t-1}}{IP_{t-1}} \times 100$ which can be approximated by $[\ln(IP_t) - \ln(IP_{t-1})] \times 100 = 100 \times \ln\left(\frac{IP_t}{IP_{t-1}}\right)$ when the change is small. Converting this into an annual (12 month) change yields $1200 \times \ln\left(\frac{IP_t}{IP_{t-1}}\right)$.

(b) The values of Y from the table are

Date	**2000:7**	**2000:8**	**2000:9**	**2000:10**	**2000:11**	**2000:12**
IP	147.595	148.650	148.973	148.660	148.206	146.300
Y		8.55	2.60	–2.52	–3.67	–7.36

The forecasted value of Y_t in January 2001 is

$$\begin{aligned}\hat{Y}_{t|t-1} &= 1.377 + [0.318 \times (-7.36)] + [0.123 \times (-3.67)] \\ &\quad + [0.068 \times (-2.52)] + [0.001 \times (2.60)] \\ &= -1.58.\end{aligned}$$

(c) The t-statistic on Y_{t-12} is $t = \frac{-0.054}{0.053} = -1.0189$ with an absolute value less than 1.96, so the coefficient is not statistically significant at the 5% level.

(d) For the QLR test, there are 5 coefficients (including the constant) that are being allowed to break. Compared to the critical values for $q = 5$ in Table 14.5, the QLR statistic 3.45 is larger than the 10% critical value (3.26), but less than the 5% critical value (3.66). Thus the hypothesis that these coefficients are stable is rejected at the 10% significance level, but not at the 5% significance level.

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(e) There are $41 \times 12 = 492$ number of observations on the dependent variable. The BIC and AIC are calculated from the formulas $\text{BIC}(p) = \ln\left(\frac{SSR(p)}{T}\right) + (p+1)\frac{\ln T}{T}$ and $AIC(p) = \ln\left(\frac{SSR(p)}{T}\right) + (p+1)\frac{2}{T}$.

AR Order (p)	**1**	**2**	**3**	**4**	**5**	**6**
SSR (p)	29175	28538	28393	28391	28378	28317
$\ln\left[\frac{SSR(p)}{T}\right]$	4.0826	4.0605	4.0554	4.0553	4.0549	4.0527
$(p+1)\frac{\ln T}{T}$	0.0252	0.0378	0.0504	0.0630	0.0756	0.0882
$(p+1)\frac{2}{T}$	0.0081	0.0122	0.0163	0.0203	0.0244	0.0285
BIC	4.1078	4.0983	4.1058	4.1183	4.1305	4.1409
AIC	4.0907	4.0727	4.0717	4.0757	4.0793	4.0812

The BIC is smallest when $p = 2$. Thus the BIC estimate of the lag length is 2. The AIC is smallest when $p = 3$. Thus the AIC estimate of the lag length is 3.

14.3. (a) To test for a stochastic trend (unit root) in ln(IP), the ADF statistic is the t-statistic testing the hypothesis that the coefficient on $\ln(IP_{t-1})$ is zero versus the alternative hypothesis that the coefficient on $\ln(IP_{t-1})$ is less than zero. The calculated t-statistic is $t = \frac{-0.018}{0.007} = -2.5714$. From Table 14.4, the 10% critical value with a time trend is −3.12. Because $-2.5714 > -3.12$, the test does not reject the null hypothesis that ln(IP) has a unit autoregressive root at the 10% significance level. That is, the test does not reject the null hypothesis that ln(IP) contains a stochastic trend, against the alternative that it is stationary.

(b) The ADF test supports the specification used in Exercise 14.2. The use of first differences in Exercise 14.2 eliminates random walk trend in ln(IP).

14.4. (a) The critical value for the F-test is 2.372 at a 5% significance level. Since the Granger-causality F-statistic 2.35 is less than the critical value, we cannot reject the null hypothesis that interest rates have no predictive content for IP growth at the 5% level. The Granger-causality statistic is significant at the 10% level.

(b) The Granger-causality F-statistic of 2.87 is larger than the 5% critical value, so we conclude at the 5% significance level that IP growth helps to predict future interest rates.

14.7. (a) From Exercise (14.1) $E(Y_t) = 2.5 + 0.7E(Y_{t-1}) + E(u_t)$, but $E(Y_t) = E(Y_{t-1})$ (stationarity) and $E(u_t) = 0$, so that $E(Y_t) = 2.5/(1 - 0.7)$. Also, because $Y_t = 2.5 + 0.7Y_{t-1} + u_t$, $\text{var}(Y_t) = 0.7^2\text{var}(Y_{t-1}) + \text{var}(u_t) + 2 \times 0.7 \times \text{cov}(Y_{t-1}, u_t)$. But $\text{cov}(Y_{t-1}, u_t) = 0$ and $\text{var}(Y_t) = \text{var}(Y_{t-1})$ (stationarity), so that $\text{var}(Y_t) = 9/(1 - 0.7^2) = 17.647$.

(b) The 1st autocovariance is

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$$
\begin{aligned}
\text{cov}(Y_t, Y_{t-1}) &= \text{cov}(2.5 + 0.7Y_{t-1} + u_t, Y_{t-1}) \\
&= 0.7\,\text{var}(Y_{t-1}) + \text{cov}(u_t, Y_{t-1}) \\
&= 0.7\sigma_Y^2 \\
&= 0.7 \times 17.647 = 12.353.
\end{aligned}
$$

The 2nd autocovariance is

$$
\begin{aligned}
\text{cov}(Y_t, Y_{t-2}) &= \text{cov}[(1+0.7)2.5 + 0.7^2\, Y_{t-2} + u_t + 0.7u_{t-1}, Y_{t-2}] \\
&= 0.7^2\ \text{var}(Y_{t-2}) + \text{cov}(u_t + 0.7u_{t-1}, Y_{t-2}) \\
&= 0.7^2\,\sigma_Y^2 \\
&= 0.7^2 \times 17.647 = 8.6471.
\end{aligned}
$$

(c) The 1st autocorrelation is $\text{corr}\,(Y_t, Y_{t-1}) = \dfrac{\text{cov}(Y_t, Y_{t-1})}{\sqrt{\text{var}(Y_t)\,\text{var}(Y_{t-1})}} = \dfrac{0.7\sigma_Y^2}{\sigma_Y^2} = 0.7.$

The 2nd autocorrelation is $\text{corr}\,(Y_t, Y_{t-2}) = \dfrac{\text{cov}(Y_t, Y_{t-2})}{\sqrt{\text{var}(Y_t)\,\text{var}(Y_{t-2})}} = \dfrac{0.7^2\,\sigma_Y^2}{\sigma_Y^2} = 0.49.$

(d) The conditional expectation of Y_{T+1} given Y_T is

$$Y_{T+1/T} = 2.5 + 0.7Y_T = 2.5 + 0.7 \times 102.3 = 74.11.$$

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