Space, Time & Kant: Three Pieces of the Puzzle

Euclid's Axioms and Postulates

- *First Axiom*: Things which are equal to the same thing are also equal to one another.
- *Second Axiom*: If equals are added to equals, the wholes are equal.
- *Third Axiom*: If equals be subtracted from equals, the remainders are equal.
- *Fourth Axiom*: Things which coincide with one another are equal to one another.
- *Fifth Axiom*: The whole is greater than the part.
- *First Postulate*: To draw a line from any point to any point.
- *Second Postulate*: To produce a finite straight line continuously in a straight line.
- *Third Postulate*: To describe a circle with any center and distance.
- *Fourth Postulate*: That all right angles are equal to one another.
- *Fifth Postulate*: That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side of which are the angles less than the two right angles.

The Clarke-Leibniz Debate (1715-1716)

An exchange of letters between Samuel Clarke, defending Isaac Newton's conception of space and time, and Leibniz, who disputed Newton's ideas.

- *Leibniz's First Argument:*
  God does not need a "sense organ" (Newton's "God's boundless uniform sensorium") to perceive objects; and space cannot be an absolute reality, or it would possess a greater reality than substances themselves. "...the postulation of an infinite, subsistent non-substance (an "unthing" as Kant later called it) is simply a monstrosity."

- *Leibniz's Second Argument:*
  Motion and position are real and detectable only in relation to other objects. Motion or position cannot be detected in relation to space itself, since space itself represents no object. Therefore empty space, a void, and so space itself, is an unnecessary hypothesis.

- *Clarke's Reply:*
  Motion is detectable in relation to space itself, for an object accelerating or rotating alone in a void betrays the effect of forces (inertial and centripetal) that exist in relation to no other object.

- *Leibniz's Third Argument:*
  There would be no reason, and so no sufficient reason, for God to create the universe one way rather than as any one of its spatial counterparts, i.e. up rather than down, right rather than left, or east rather than west. Therefore, spatial relations are symmetrical relations among objects that are equivalent and do not exist apart from objects.

- *Kant's Reply:*
  Asymmetrical objects and their mirror-imaged counterparts (i.e. right-handed and left-handed "incongruous counterparts") are genuinely and physically different. No rotations in three-dimensional space, e.g. of right and left hands, can turn one into the other. Since the objects differ only in their spatial relationship--i.e. they could be rotated into each other through a fourth spatial dimension--they reveal that space itself is real and independent of the objects.

Kant's Theory of Space and Time

- *Ontology:*
  Kant postulates that space and time do not really exist beyond human experience, but are "forms of intuition" (i.e., conditions of perception, imposed by our own minds). This enables him to reconcile Newton and Leibniz: agreeing with Newton that space is absolute and real for objects in experience (i.e., for phenomenal objects open to science), but agreeing with Leibniz that space is really nothing in terms of objects as they exist apart from us (i.e., with things in themselves).

- *Epistemology:*
  Unlike Hume, Kant denies that the axioms of geometry are self-evident or true in any logically necessary way. They are logically "synthetic," which means that they may be denied without contradiction. That is a significant claim because it implies that consistent non-Euclidean geometries are possible (involving the otherwise consistent denial of one or more of the axioms of Euclid, as Bolyai, Lobachevskii and Riemann actually accomplished). Nevertheless, Kant holds that the axioms of geometry are known *a priori* (i.e., that they are known to be true independently of any experience) because Euclidean axioms depend on our "pure intuition" of space, namely space as we are able imaginatively to visualize it. Only if non-Euclidean space can be visualized would Kant be wrong.

- *Cosmology:*
  Kant does not think we can know, or even imagine, the universe as either finite or infinite, in space or in time, because space and time are only forms of perception and cannot be imagined or visualized as absolute wholes. The universe, as the place of things-in-themselves, is not in space or in time and so is neither finite nor infinite in space or in time. Thus there cannot be an *a priori*, rational or metaphysical, cosmology.

General Relativity: Space and Time after Einstein

- *Ontology:*
  Kant was wrong: space and time really exist beyond human experience, but only relative to masses in motion (there is no absolute, Euclidean metric to which all physical events conform: space curves locally and times are desynnchronized for objects moving in non-uniform inertial frames).

- *Epistemology:*
  Kant was wrong: non-Euclidean space can not only be visualized, but measured (the sun, for example, warps local spacetime by approximately four seconds of arc per century)--suggesting that Kant had the relation between what can be conceived and what can be visualized backwards.

- *Cosmology:*
  Kant was wrong: although the First Antinomy purports to show the impossibility of conceiving the universe as either finite or infinite in-itself (because both contradictory metaphysical absolutes can be argued and justified with equal force, it follows that neither can actually be proven), Einstein answered Kant by proposing a consistent non-Euclidean (Riemannian) universe that is finite but unbounded (i.e. without an edge).