Remote Learning Module for 10 April 2020

Lecture Notes on *Mind, Matter, and Mathematics* – Chapter 4

Last class we examined the foundations of mathematics, delving into some of the finer points that distinguish among Realism, Intuitionism, and Formalism. We then considered some of the implications stemming from the "unreasonable effectiveness of mathematics" for carrying out the everyday business of natural science, with reflections on Einstein and Bohr. In the course of their conversations on these topics, we found Changeux and Connes primarily concerned with metaphysical issues, that is, worries about the nature and existence of mathematical objects. Today we'll turn our attention to the epistemic side of their controversies—the nature and justification of mathematical judgements. We've seen most of the neuroscience discussed in this chapter in other contexts, so for today, let's be sure to focus on the application of Changeux's "levels of organization" hypothesis in relation to Kant's epistemology.

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(1) Illumination.

We begin with an observation by Changeux that mathematicians, by and large, do not take much of an interest in brain-science. We might add: neither do mathematicians look to computer algorithms (including machine learning) for much help in their work. We can see in this a skepticism as old as Plato's original distrust of the use of images to stand in for ideas—a theme that runs throughout his dialogues. Note also that as the chapter opens, we meet the figure of Hippolyte Tain, the early positivist whom we encountered in Hacking's discussion of *dédoublement* during the 19th century, and whom Changeux holds up as far superior to the likes of Sartre and Foucault. In any case, they go on to take Jacques Hadamard's *Psychology of Invention in the Mathematical Field* as the point of departure for their reflections on the nature of mathematical thinking. In Hadamard we find the following cognitive model: there are four stages involved in the psychology of mathematical discovery: (i) Preparation, (ii) Incubation, (iii) Illumination, and (iv) Proof.

Connes prefers three stages, collapsing Hadamard's first two stages into one. He further supposes that moments of illumination often engage an emotional response—when an oblique strategy can be seen to promise a solution to a problem that is difficult to approach head-on. What happens, he thinks, amounts to a Darwinian metaphor: like the Tower of Generate and Test, mathematical insights follow after a range of possible alternatives are shuffled and re-shuffled until a key is found to fit a lock, whereupon an emotional trigger (in the form of a pleasure alarm) is released. Here, we might recall Damasio's contention that pain responses are more functionally relevant to practical decision making than pleasure responses. If so, we can perhaps use this observation to support Connes's earlier remarks about the difference between physics and mathematics: pain for the material world; pleasure for the conceptual world.

(3) The Brain & Its Multiple Levels of Organization.

Citing the failure of spontaneous generation as a theory emerging from the rejection of vitalism, Changeux finds that Levels of Organization should best be thought of a Natural Kinds (not just arbitrary, however useful, tools)—levels that emerge from causal relations between structure and function (a theme we also saw in Churchland). Changeux finds Kant's distinctions between the faculties of Sensibility, Understanding, and Reason to be on a par with Connes' faculties of Calculation, Reflection, and Discovery. In either case, these faculties or powers can be seen as functions; so they ask: What structures cause them to emerge as cognitive events.

According to Changeux, our brains display six levels of structure. The first three are simple enough: (i) Atomic (the building blocks), (ii) Molecular (neurotransmitters), and (iii) Neurons (structures capable of storing and releasing electromagnetic charge). The last three, Changeux thinks, are the very levels of structure from which Kant's faculties of human cognition emerge.

Level of Organization	Kant's Faculty Psychology
(iv) Neuronal Circuits	Forms of Sensibility (Time & Space)
(v) Neuronal Assemblies	Concepts of the Understanding
(vi) Assemblies of Assemblies	Principles of Reason

Connes, on the other hand, prefers a simpler set of associations.

Level of Organization	Connes' Facutly Psychology
(iv) Neuronal Circuits	Calculation
(v) Neuronal Assemblies	Reflection
(vi) Assemblies of Assemblies	Discovery

On either model, Changeux and Connes find that they at least agree on a distinction between the tactical and the strategic uses of reason. Tactical deployment is regulated by classical logic and involves observed relations among causes and effects. Strategic deployment is not bound by classical logic and involves fitting means to ends.

Recalling our discussion of processing speed in Dennett's *Kinds of Minds*, note that here we find the assertion that the (relatively slow) absolute speed of neuronal processing at the higher levels of brain function are attributed to the same piggy-back engineering we saw in Damasio: the higher functions utilize the same structural components that phylogenetically earlier organisms added to the tower of generate and test. We might go so far as to assert that this principle of piggy-back engineering applies even at the atomic level. For example, consider that the functionality of the DNA molecule depends on the *weak* hydrogen bonds that hold the base pairs (A,T) and C,G) together.

(4) From Elementary Circuits to Mental Objects.

Tracing evolutionary history from the first macromolecules to human brains, Changeux illustrates the Vector Hypothesis with the example of a monkey's brain, where we find mapping functions for spatial location subserved by a brain analog to the Cartesian coordinate system.

Changeux thinks that the neuronal circuits involved here represent the emergence of concrete empirical representations, and so he supposes these nicely ground Kant's Concepts of the Understanding in a firm neurobiological anatomy. However, we should be cautious here. Note that, for Kant, the Concepts of the Understanding (for example, the Categories of Relation: substance/attribute, cause/effect, and agent/patient), although they are certainly structured:

(a) are *empty* in themselves;
(b) require *sense-data* in order to posit empirical objects (that is, to represent things); and
(c) constitute a set of *rules* for generating the *experience* of objects.

Nevertheless, note that these rules, suitably adjusted, can serve rather nicely as grounds for the possibility of the experience of mathematical objects as well, and if so, Changeux has a good warrant for adopting Kant's model.

In fleshing out this model, we also find ourselves on familiar ground. Just as we found in Damasio's somatic marker hypothesis, Changeux and Connes find that:

(a) the use of brain lesions and localized dysfunction helps advance the construction of models whereby we can dissect mathematical faculties with reference to distinct levels of organization in brain function; and

(b) the prefrontal cortex is critical to all forms of hypothesis formation and anticipating the future.

Thus, we can localize numerical alexia or agraphia to the left parietal lobe, while spatial acalculia is localized to the right somatosensory cortex. Also, in keeping with the somatic marker hypothesis, we find the implication that the relevant frontal regions that are compromised in "utilization behavior" (the indiscriminate use of objects without regard to their intended purposes) are the very regions that normally produce the *relevance criteria* needed to solve complex problems.

(5) Generalized Darwininsm.

Changeux now presents a variant of Popper's evolutionary epistemology: in brain development we find two components necessary for emergent levels: (i) a Diversity Generator, and (ii) a Selection Mechanism. From these two components, new forms can emerge from already *structured* elements. In brief: the brain is a "Darwin Engine." The basic model, Changeux adduces has the following ingredients:

Matter → Variation → Form → Function
↑______________________|

We'll find another version of Generalized Darwinism in Calvin's *Cerebral Code*:

Pattern + Copies + Variations + Selection + Reproduction = Evolution

On either model, we can trace an evolutionary epigenesis in brain development: the same process of that drives the evolution of biological species is recursively replicated in a mathematician's brain, thus producing mathematical objects. This "Mental Darwinism" is also of a piece with Damasio's somatic marker hypothesis. What happens in the course of mathematical discovery is:

(a) pre-representations are coughed up and compared with the world;
(b) best-fit congruence occurs via harmonic resonance among active neuronal assemblies; and
(c) the best fit becomes stabilized.

(6) Mental Darwinism and Mathematical Creativity.

Connes returns to the discussion wielding a vengeful Pythagorean sword: it is the inexplicable coherence of mathematical reality that permits neuronal assemblies to achieve collective harmonic resonance; neuronal assemblies, in other words, produce satisfactory experiences when they match up with the structure of mathematical reality.

Changeux's riposte, however, is to return to a recognition that mathematics has a history, and that this history involves (a) *beliefs* (elements that are held immune from revision by doctrinal practice), on the one hand, and (b) *hypotheses and conjectures* (falsifiable claims, subject to skepticism or reasoned consensus) on the other hand. Consensus, on his view, is little more than the emotional response which occurs when a proof marks the key to a lock invented by a culture of mathematicians. On this view, the internal coherence of mathematical objects is an equilibration that is built up over time; we experience a charge of pleasurable emotion when we are able to anticipate the agreement of an intellectual community.

At this point, Connes demurs, insisting that this view cannot be right since it casts philosophical doubt on the real existence of numbers. And so, we are at stalemate.

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Next time, we'll look to Chapter 5: "Darwin among the Mathematicians." Be well everyone, and, although I imagine you are probably quite tired by now of my continuing to say so, do remember: social distancing continues to save lives, which is presumably why we are still not in JUB 202 presently.