Remote Learning Module for 13 April 2020

Lecture Notes on *Mind, Matter, and Mathematics* – Chapter 5

Last class we considered the hypothesis that the levels of organization in human brains, ranging from the atomic level to the level of assemblies of assemblies of neural circuits, might instantiate the cognitive elements involved in the process of mathematical discovery. Along the way, we saw Changeux positing a similar project for mapping Kant's cognitive psychology (sometimes called a kind of transcendental machinery whereby sense data are filtered and schematized into the experience of empirical objects). In the course of this conversation, we found that the centerpiece of this hypothesis (relating structure to function) is the postulation that thoughts form up in the brain in exactly the same way that species proliferate in an environment, that is, by the evolutionary process of variation and selection. Today we look deeper into the Darwinian schema with our primary focus trained on neurological correlates to natural selection.

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(1) Utility of the Darwinian Schema.

In biological evolution we think of selection effects as issuing from the struggle for survival: those variations that enable a species better to compete for resources, and to adapt to environmental vicissitudes, than other variations, will proliferate until they eventually dominate their habitats. Changeux and Connes agree that it is at the third level of mathematical discovery—illumination or insight—that we should look for neural analogs to biological selection effects, because if Darwinian reduction works here, it will work anywhere.

Recall that, for Neural Darwinism, the selection of favorable variations requires an *Evaluation Function* to perform the selection. Both Changeux and Connes like the notion of a chaotic range of circuits settling down to a stable harmonic resonance as characterizing the kind of neurological event that will serve as an evaluation function in mathematical thinking. But whereas Connes supposes a resonance between the activity of neural assemblies and real mathematical objects (archaic mathematical reality), Changeux takes up Ockham's Razor again, supposing that the requisite resonance can be found between a new thought and existing knowledge held in the long term memory of a given mathematician.

On either view, however, our intrepid interlocutors consider the variations over which a given selection effect ranges to be instances of *analogical reasoning*: multiple patterns of similarity, only one of which emerges as optimal at the moment of illumination.

(2) Coding Stable Forms.

Returning to the relation between structure and function (neuroanatomy and neurophysiology, respectively), they ask: What mechanisms are needed to accomplish Long Term Potentiation (LTP). Connes thinks that whatever the mechanism may be, from a logical point of view, it has to be hierarchical. But from a physiological point of view, the best model, he thinks, will turn out to come from topology rather than geometry. And not just any topology, but the specific topology of *simplicial complexes*. Why such complexes? Because (a) they can be found active in actual brain events, and (b) they are functionally equivalent to hierarchical trees.

Simplicial complexes arrange points (vertices) and lines (edges) in n-dimensional spaces wherein we can classify sets of points/lines by a Compatibility Rule: the Euler-Poincaré Characteristic, which is equal to V – E + T (that is, vertices minus edges plus triangles). According to Connes, it is this Compatibility Rule that establishes the right sort of homeomorphism between brain events and cognitive events. Changeux applauds this rule as well, noting that it makes good sense of how we manage insight in moments of facial recognition.

What Connes finds most fertile in this model is that it provides a ready way for topological structures to achieve unity from diversity: the same topological object can assume many different realizations. The pay-off here that of revealing the very structure of LTP (that is, revealing how memories are organized in assemblies of assemblies of circuits). In particular, Connes contends that memories are realized in brains by way of Hyperbolic Simplicial Complexes; he thinks so because these complexes can be shown to serve exactly the same function as a hierarchical tree, but in n-dimensional space rather than the one-dimensional organization of lines ascending or descending a tree. One easy way to grasp what's going here is to consider the two different ways you have for moving the cursor on your computer screen (you are staring at a computer screen right now, no?): using your arrow keys or using your mouse. Mice move on a geodesic, while arrow keys move along branches, one step at a time. If you miss a branch using your arrow keys, you'll have to back down until you find the requisite node where the appropriate branch begins; but with the mouse, all you have to do is get close with one swooping gesture, and then hone in on the exact spot: the mouse is both more efficient and more effective.

There is a guiding principle governing this model which neither Changeux nor Connes notices: we must assume that nature will always obey what the physicist, Richard Feynman, called the Principle of Least Action: natural phenomena will follow paths of less resistance because such paths require less effort.

(3) Reasoning by Analogy.

Analogical reasoning would seem to involve two stages: (a) Detecting the existence of an analogy (a set of similarity relations) in the first place, and (b) Distinguishing relevant from irrelevant similarities by following three steps:

(i) Replication: whereby we reproduce features of the original in the copy;

(ii) Transmission: whereby we replace the some but not all of the reproduced features with new ones; and

(iii) Amelioration: whereby we test the analogy for rightness-of-fit.

(4) Mathematical Darwinism.

In order to link representations within frameworks of thought, they agree that we must be able to answer two further questions:

(a) How do *conceptual* links produce *sentences* we can test?

(b) How are intentions (purposes) defined in mathematics?

Connes considers that we best answer the second question—the question of intentions—by specifying an Evaluation Function (he means a mechanism for telling us how far we stand from a goal given in advance). If so, Changeux further considers that we should expect this sort of function to work by way of harmonic resonance in phase space—that is, by constructive or destructive interference of frequencies of brain waves. Connes reminds him that this is little more than a functionalist promissory note; how actual brains accomplish this sort of resonance remains unclear. Changeux replies *à la* Damasio: look to the limbic system: hypotheses are driven by desire.

(5) The Natural Selection of Mathematical Objects.

Changeux sums up his position: if we take Darwinism seriously (for modeling moments of mathematical illumination), then we should posit a combination of already existing elements (belonging to established math) giving birth to something new.

Connes sums up his position: moments of illumination occur when there is a coherence between neural events and real, independently existing mathematical objects, so that all selection effects must in the end reduce to the Principle of Contradiction. Reasoning amounts to recognizing how one would solve a problem, but insight occurs only at the point when one comprehends the solution instantly.

Before we close for the day, let's take another look at question (a) above. Perhaps they've framed this question wrongly; perhaps instead of asking how conceptual links produce sentences, we should be asking how do the sentences we say when answering questions produce the mental objects we call concepts? It may even turn out that there are no such things as concepts located in the neural machinery of our brains; it may be that there are only sentences after all.

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Next time, we'll look to Chapter 6: "Thinking Machines," or the prospect of engineering Artificial Intelligence. Here, we'll consider three flavors of theory: (i) GOFAI (Good, Old Fashioned Artificial Intelligence), (ii) Neurocognitive Science, and (iii) Neuromimetics. Be well everyone, and, although I imagine you are probably quite tired by now of my continuing to say so, do remember: social distancing continues to save lives, which is presumably why we are still not in JUB 202 presently.