The EPR Paradox, Gun & Camera in Hand

I. Philosophical Import: Since the 1927 Solvay Conference, the Bohr-Einstein Debates focused on the metaphysical implications of QM:

- **Realism** (Einstein's view): scientific laws are true just in case they match objective reality
- **Instrumentalism** (Bohr's view): scientific laws are true just in case they allow us to formulate a coherent, self consistent, systematic picture of reality, integrating all the data available.

II. Conception & Apparatus:

(a) The RC (reality criterion): if, without disturbing a system, we can predict, with certainty (P=1) the value of a physical quantity, then there is an element of reality that corresponds to that quantity.

(b) Construct a wave function, $\psi$, that determines the correlation between two particles: it measures:

:: their relative distance
:: the sum of their momenta

(c) We cannot measure the momentum of either particle separately, but we can measure the screen before and after collision, so that from the recoil we can calculate the total momenta of (I+II).

(d) Thus, there are two physical quantities that have definite values, and so correspond to real physical attributes; but the position of each particle is undetermined (exact location of the slits cannot be known at the passing point); neither can the individual momenta be determined. **On QM neither the position nor the momentum** $(q,p)$ of either is real, but as a pair, they have both.

(e) After separation of the systems, we can measure the position of one without disturbing the other, so we can know its position (or momentum if we choose).
(f) But now we see that without disturbing particle 2’s position or momentum, we can predict for 2 either q or p! This means that our choice of what to measure as regards the first particle determines the real properties of the second instantaneously.

(g) If QM were complete, it would mean that the uncertainty principle affects position/momentum measurements on separated particles, in which case the particles must still be exerting an influence on one another—a clear violation of Einstein Separability.

III. Analysis:

(1) Let the Schrödinger wave function, \( \psi \), characterize the state of a system, and let the question of the completeness of quantum mechanics (QM) be the question, "Are there physically real properties of the system not represented by \( \psi \)?"

(2) Let 'A' be an operator corresponding to the observable property, momentum, such that \( A\psi = a\psi \) (where the eigenvalue 'a' corresponds to the numerical value received by measuring the momentum corresponding to A for a system in state \( \psi \)). Similarly, let 'B' be an operator corresponding to the observable property, position.

(3) Since \( A \) and \( B \) are, according to QM, noncommuting operators, there will be no eigenvalue equations relating \( A \) to \( B \), and yielding one eigenvalue. Only a probability can be computed, e.g.:

\[
P(a,b) = \int_a^b \overline{\psi} \psi \, dx = b - a
\]

(4) Now, let two systems, I and II, interact between \( t=0 \) and \( t=T \), and assume that the state of I and II are known before \( t=0 \).

(5) Let \( \psi \) stand for \( I + II \) (N.B., no information about either I or II, taken separately, is possible).

(6) Consider two possible expansions of the wave function, \( \psi \), for \( I + II \):

\[
\begin{align*}
(6.1) \quad \psi(x_1,x_2) &= \sum_{n=1}^{\infty} \theta_n(x_2) \mu_n(x_1) \\
(6.2) \quad \psi(x_1,x_2) &= \sum_{s=1}^{\infty} \phi_s(x_2) \nu_n(x_1)
\end{align*}
\]

(I.e., a particle, 1, in system I has \( \mu_1(x_1), \mu_2(x_1), \ldots, \mu_n(x_1) \) as eigenfunctions with eigenvalues \( a_1, a_2, \ldots, a_n \) for the momentum quantity, and \( \nu_1(x_1), \nu_2(x_1), \ldots, \nu_n(x_1) \) as eigenfunctions with eigenvalues \( b_1, b_2, \ldots, b_n \) for the position quantity.) The relevant expansion coefficients are therefore: \( \theta_n(x_2) \) and \( \phi_s(x_2) \).

(7) Now, if a measurement of \( A \) for particle 1 yields the value \( a_k \), then 1 is in the state characterized by \( \mu_k(x_1) \). Therefore, the remaining coefficients vanish, so we can compute that particle 2 in system II is characterized by \( \theta_k(x_2) \) with the result that a measurement performed only on 1 yields values for particle 2 without disturbing system II after separation.