Background Assumptions:
Newton, Einstein, and the Whole Truth about Energy

[After L.C. Epstein's demonstration, "For teachers only," in *Relativity Visualized* (Insight Press, 1991), pp. 125-6.]

1. Newton's second law of motion reads, "the change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed" (*Principia*, L2). Since Newton defines "quantity of motion" as "arising from the velocity and quantity of matter conjointly" (*Principia*, D2), the second law yields

$$f = ma = m(dv/dt).$$

2. But tucked into the folds of the second law lies an implicit assumption: the invariance of mass. The problematic character of this assumption does not emerge among phenomena occurring at (substantially) subluminal velocities. For example, the second law correctly yields the classical kinetic energy equation:

Since	(a)	energy is *force* times *distance*, an energy increment is $dE = f\,dx$.
So, if	(b)	$f = m(dv/dt)$ and $dx = v\,dt$,
then	(c)	$dE = (m\,dv/dt)(v\,dt) = m\,v\,dv$,
integrating:	(d)	$E = mv^2/2$.

3. But the second law isn't the whole truth; emended to avoid the assumption of mass invariance, it should read:

$$f = m(dv/dt) + v(dm/dt)$$

which means that even if *acceleration*, *dv/dt*, is zero, the *force* need not be zero if the *mass* of a moving object changes with time.

4. Now, since the speed of light is constant, for light *dv/dt* is always zero. So, for light the force equation becomes:

$$f = v(dm/dt).$$

When this force is put into the energy increment equation, it yields:

$$dE = (v\,dm/dt)dx = (v\,dm/dt)\,v\,dt.$$

The *dt* terms cancel, of course, and since the *v* here is *c*,

$$dE = c^2 dm,$$

integrating

$$E = mc^2$$

which is the whole truth about energy.