# Remote Learning Module for 30 March 2020

Lecture Notes for Fernando Espinoza's *The Nature of Science,* Chapter 2

Last time we examined the logic of paradoxes, finding that a paradox can be resolved by showing that (a) it follows from one or more false assumptions, (b) only one of the apparently incompatible beliefs is warranted, or (c) the two beliefs are actually compatible, once we clarify the meanings of the terms involved. We also found that some paradoxes appear to resist resolution, primarily, though not exclusively, when the paradox involves some sort of self-reference, as, for example, in the sentence: "You do not believe this sentence."

At the end of class we faced the "paradox of the pop quiz," in which we imagine a teacher informing a class that "One day this week, class, I will give you a pop quiz *but* you will not it until that day." The students reasoned that this would be impossible by eliminating each day of the weeks backwards from Friday. But on Wednesday they received the pop quiz. We asked: What error did this students make in their thinking? The answer is that the students confused knowing with believing.

Today we'll turn our attention to the rest of Chapter 2: Origins of Accomplishing Tasks.

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**(1) The Development of Scientific Knowledge.**

We begin with the earliest known use of *tools*, noting that we can classify tools into three categories: hand tools, machine tools, and mind tools. These categories are not exclusive, though. For instance, taking arithmetic to be a mind tool, we might think of an abacus as both a mind tool and a hand tool, while an electronic calculator would be both a mind tool and a machine tool.

**(2) The Earliest Tools.**

Primates like Chimps and Orangutans have been observed to use tools to secure food, as have Ravens; but Hominids appear to be the first to manufacture tools for the purpose of *making better tools*. Of special interest in this regard is the evidence from *stone flake* tools, which reveals that humans developed the differential handedness we see among us today (87% Right-handed compared to 13% Left-handed) between 1.9 and 1.5 million years ago, before which, as in other mammals, the differential was approximately 50/50). While many explanations of this phenomenon trade on lateralization of brain functions, it is possible that we are looking at one of the first instances of the influence of *public education* on individual behavior: follow the leader.

On pages 15-21 in our text you'll find a nice table tracing the chronology of life on Planet Earth. From this chronology we can also gain perspective on the rise of us tool-users:

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|---|---|
| - The oldest hominid tools | - about 2 million years ago |
| - Fire | - 800,000 years ago |
| - Neanderthals | - 400,000 years ago |
| - Cave art | - 35,000 years ago |
| - The needle | - 20,000 years ago |
| - Domestic animals | - 10,000 – 6,000 BCE |
| - The Bronze Age | - 3,000 BCE |

Perhaps the first machine tool was the water wheel (developed in Mesopotamia around 200 BCE.

**(3) The First Mind Tools.**

Long before the water wheel, however, humans developed mind tools like calendars and alphabets. We have found lunar notations inscribed on bones as early as 35,000 BCE; the Phoenician alphabet dates from about 900 BCE. These mind tools display the expanding sphere of human ambitions: from calendars to predict planting seasons to incantations to petition the gods, and from laws to govern human behavior to the laws of proportions that govern the natural world.

The ancient Babylonians invented a sexagesimal number system to map the ecliptic (that is, the daily path of the sun) and thus the zodiac. We are all familiar with this sexagesimal (base-60) system; we use it to keep track of the minutes in an hour on a standard analog clock. If you play tennis, you may have noticed that scoring points in this game deploys the same system, with a point made at each quarter-hour.

The Babylonians also developed a system of elementary algebra for solving geometrical problems, having determined, for instance that $(a + b)^2 = (a^2 + b^2 + 2ab)$. These algebraic schemes, however, were heuristic, that is, they were *rules of thumb*. The Babylonians also employed the rule we know today as the Pythagorean Theorem long before the figure of Pythagoras was born. Tradition tells us that Pythagoras coined the word "philosophy" from two Greek words: *philia* + *sophia*. "*Philia*" is the word the Greeks used for the love between friends (the city of Philadelphia was so named to be known as the "city of brotherly love." And "*sophia*" is the term we usually translate to English as "wisdom." So, the *philosophers*, for Pythagoras and those who came after him, were "the friends of wisdom," and *philosophy* was the "love of wisdom." Moreover, it was Pythagoras who transformed Babylonian rules of thumb into the idea of *proof*, and this was a decisive turn in the progress of Western science.

**(4) Pythagoras.**

We find in the Rhind Papyrus that the ancient Egyptians not only approximated the value of *pi*, but entertained themselves with a wind range of mathematical riddles. All the same, these were still rules of thumb. It was Pythagoras (582-500 BCE) who first introduced the idea of *Rigorous Proof*. The features of rigorous proof are two-fold: (a) the proof must proceed stepwise, with each step warranted by observing that its negation would be or would lead to a contradiction; and (b) the proof must be public.

The figure of Pythagoras is rather shadowy; some scholars have even suggested that there was no such historical person, but rather that the name was invented by a secret mathematical society so as to imagine having a single illustrious predecessor. In any case, the ancient historians certainly took him to have been a real person. They believed him to have been born on the island of Samos in Ionia. He was thought to have travelled extensively throughout Babylonia and Egypt (where he learned mathematical technique); and he was said to have had an encyclopedic memory. When he was somewhere between 30 and 40 years old, Pythagoras travelled to Italy (first to Croton and then to Metapontum), where he formed a school of thought.

The idea of memory is, as it turns out, of special significance in the legends surrounding Pythagoras and his school. We learn from the historian of philosophy, Diogenes Laertius, that among the doctrines Pythagoras taught his followers, there was a strong commitment to vegetarianism. The reason for this practice, according to Diogenes, was because to eat the flesh of an animal was possibly to be eating the flesh of one's own ancestors. And the reason for this possibility was the doctrine of *metempsychosis*, which means, "travelling souls." You might think of this as kindred to the idea of reincarnation, but it is slightly different. The Pythagorean notion of *metempsychosis* came from an earlier myth in which a mortal man, Aethalides, having done a favor or the god, Hermes (messenger of the gods) requested in return that Hermes grant him (Aethalides) immortality. Unfortunately, Hermes was not able to grant this wish since the only difference the Greeks to divide the gods and humankind was immortality. And so, Aethalides asked Hermes for what the thought to be the next best thing: Eternal Memory (instead of divine immortality). Hermes was able to grant this wish: Aethalides' memories would thereafter travel to a new person each time the old one died. And Pythagoras claimed to have been born into this lineage.

**(5) The Musical Scale.**

The Pythagoreans held that all reality has a mathematical structure, and that such structures appear among integers and ratios of integers. Pythagoras was thought to have identified one of these structures in the consonances (or pleasing combinations) found in Greek music: the octave, the fourth and the fifth. The respective ratios were 2:1, 4:3, and 3:2. According to the legend, as Pythagoras was passing a blacksmith's shop one day, he heard these consonances emerging from the hammering within. He then commandeered the blacksmith's hammers and compared their weights, finding them to line up in the series 12 – 9 – 8 - 6 units. Simplifying the ratios among these weights we find that 12:6 yields the octave, 9:6 yields the fifth, and 8:6 yields the fourth.

Centuries later, in 1636, the French mathematician, Marin Mersenne, published a work entitled *Harmonie Universelle* (or Universal Harmony) in which he demonstrated that by attaching a series of different weights to a string and striking the strings, the resulting tones would yield notes in the musical scale. Mersenne also demonstrated that the differences among these tones was a function of the frequency with which the string vibrated back and forth. From these observations, Mersenne was able to conclude three laws: that frequency of vibration is (a) proportional to the square root of string tension, (b) inversely proportional to string length, and (c) inversely proportional to the square root of string thickness (that is, its density). These results show that Pythagoras simply *could not have used the weights of hammers* to have determined the numerical ratios 2:1, 4:3, and 3:2 for string lengths on a monochord. Can you see why?

**(6) The Problem of Incommensurables.**

The term, "incommensurable," means "having no common measure." Applied to geometry, we say that two magnitudes, two line lengths, are incommensurable when their ratio cannot be expressed as a ratio of integers, for example, the ratio of a circle's diameter to its circumference, that is, the value of *pi*.

As we just noted, Pythagoras and his disciples held that nature is fundamentally mathematical, which is to say, differences among quantities explain differences among qualities. They also held that mathematical truths express relations among integers. The combination of these two doctrines led to a crisis for the Pythagoreans. In particular, one of these Pythagoreans, Hiappasus of Metapontum, made public a proof that the diagonal on the unit square is *not* a ratio of two whole numbers—that is, the diagonal of a square whose sides measure 1 unit, cannot be expressed as a ratio of any two whole numbers, m/n. These two magnitudes have not common measure. Poor Hiappasus: for publishing his proof, the Pythagoreans expelled him from their society, took him out to sea, and drowned him. What Hiappasus did was generate a paradox from the following four propositions, not all of which can be true at once.

(i) Reality has a mathematical structure.
(ii) If so, then physical phenomena can be expressed in numbers.
(iii) The numbers are integers or ratios of integers.
(iv) The diagonal on the unit square is incommensurable with its sides (is *not* a ratio of integers).

This paradox was not fully resolved until 1872, when the Irrationals were shown to be Real Numbers after all (in the work of Weierstrass, Cantor, and Dedekin).

**(7) Pythagorean Triplets.**

Pythagorean triplets are three whole numbers, a, b, c, that satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. Here is a nice algorithm for generating Pythagorean triplets for every odd number:

Let a = any odd number, then
Let $b = (a^2 – 1) / 2$, and
Let c = b +1

This works because the differences between successive square numbers are successive odd numbers. Can you think of an algorithm that will yield a Pythagorean triplet for every even number?

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Next time, we'll turn our attention Chapter 3 – What is Rational Inquiry? In this chapter we'll meet Anaximander again, along with several more of the first natural philosophers of ancient Greece. Be well everyone, and remember: social distancing saves lives, which is presumably why we are still not in JUB 202 presently.